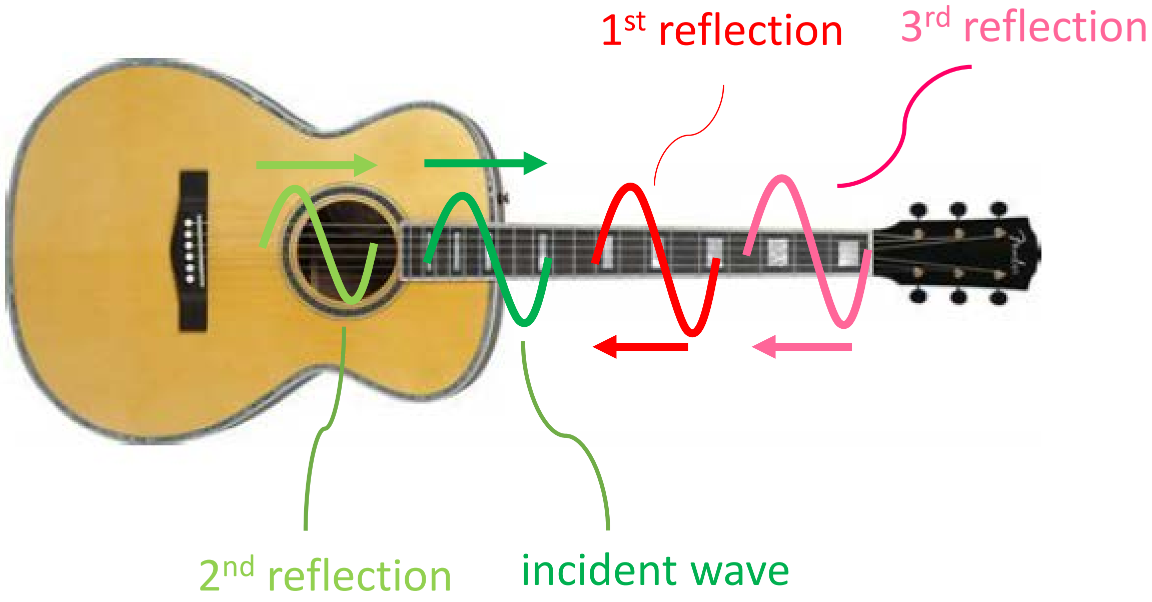
D.2 Single Source Interference: Standing Waves

When you pluck a string, or blow on the end of a clarinet, or flute, you create a wave in the string/air column, and this wave will reflect back and forth (and be partially transmitted into the guitar/outside air too) along the string/air column. A few reflections are illustrated.



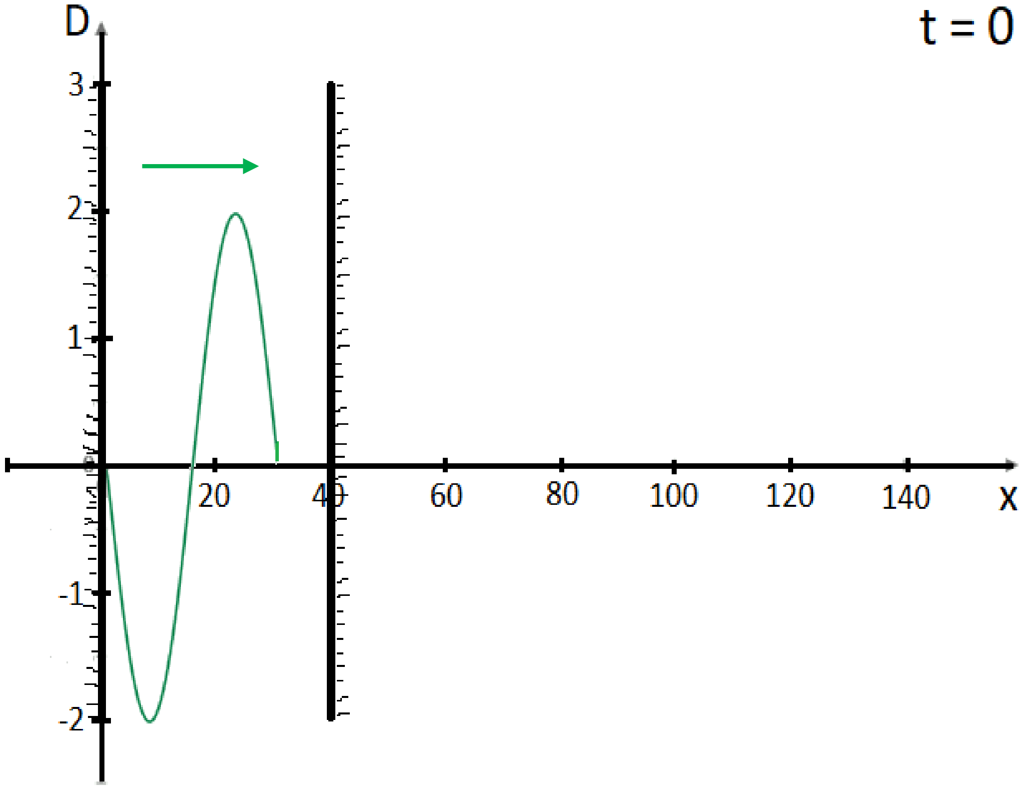
We're ultimately interested in how these multiple reflections superimpose….

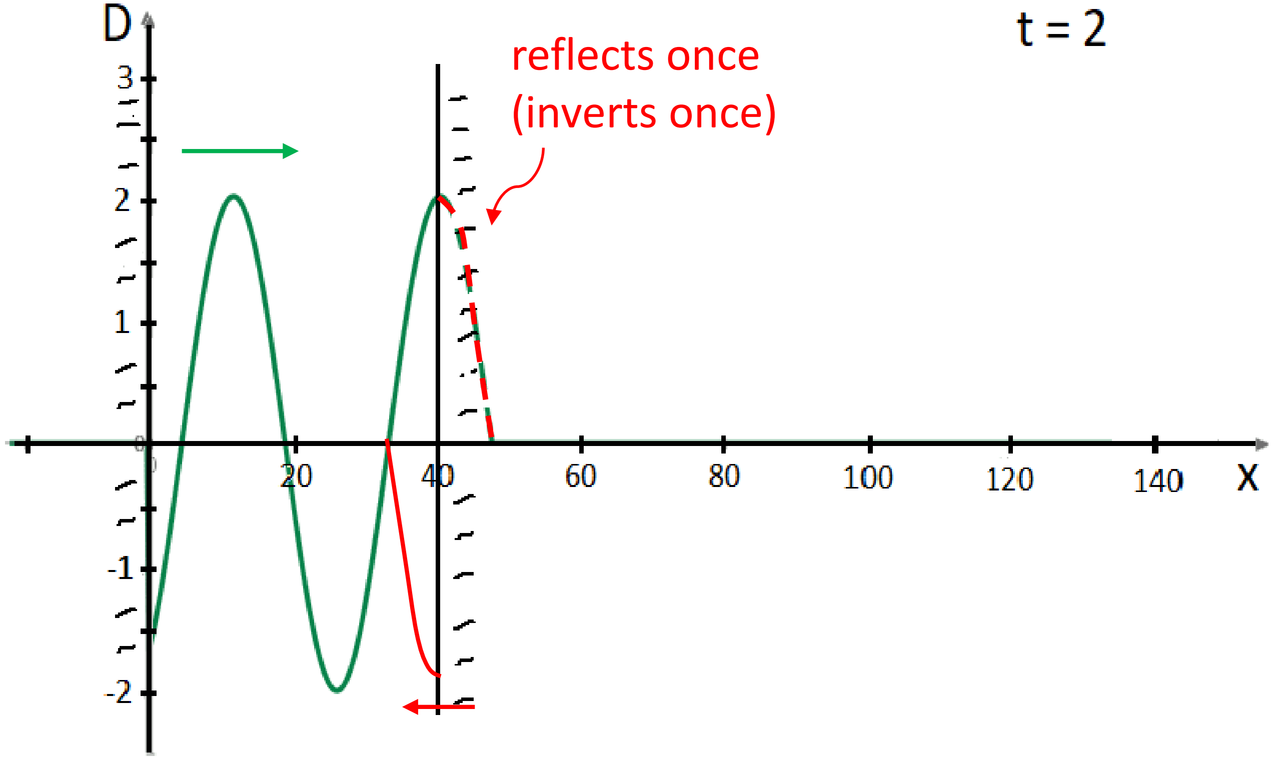
A wave is said to 'resonate' when it and its reflected waves all meet the following criteria.

1. All waves traveling to the right superimpose exactly on top of each other.
2. All waves traveling to the left also superimpose exactly on top of each other.

D.2 Single Source Interference: Standing Waves

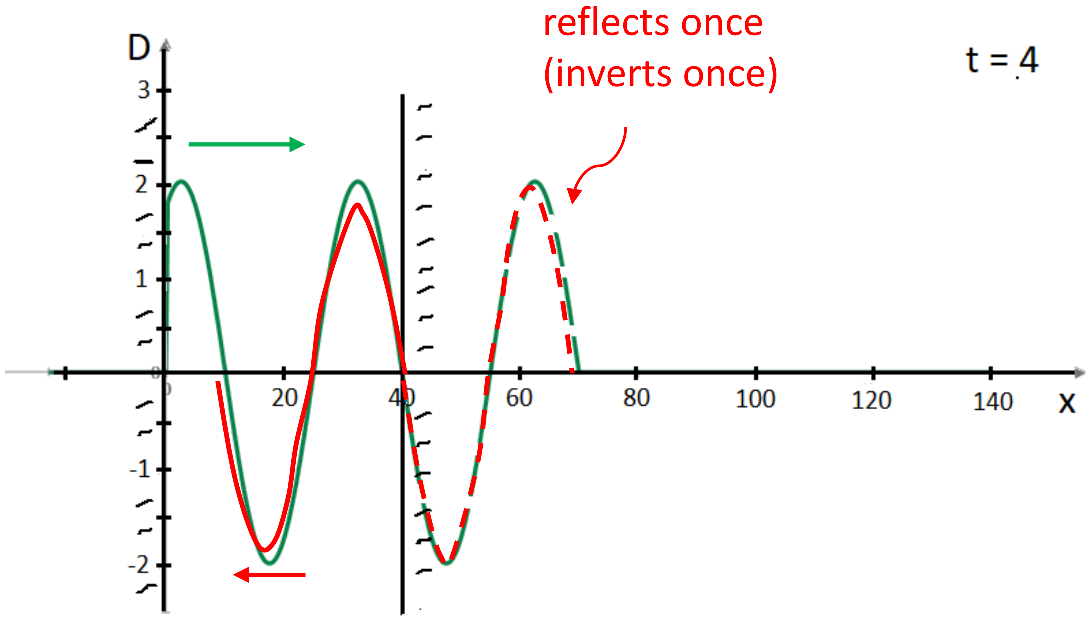
Most of the time, this criteria will not be met. Consider the following case: a λ = 30cm long wave traveling down a 40cm guitar string at the contrived velocity of 10cm/s. A guitar string's boundaries would be considered to be 'hard' and so the wave will *invert* upon every reflection.

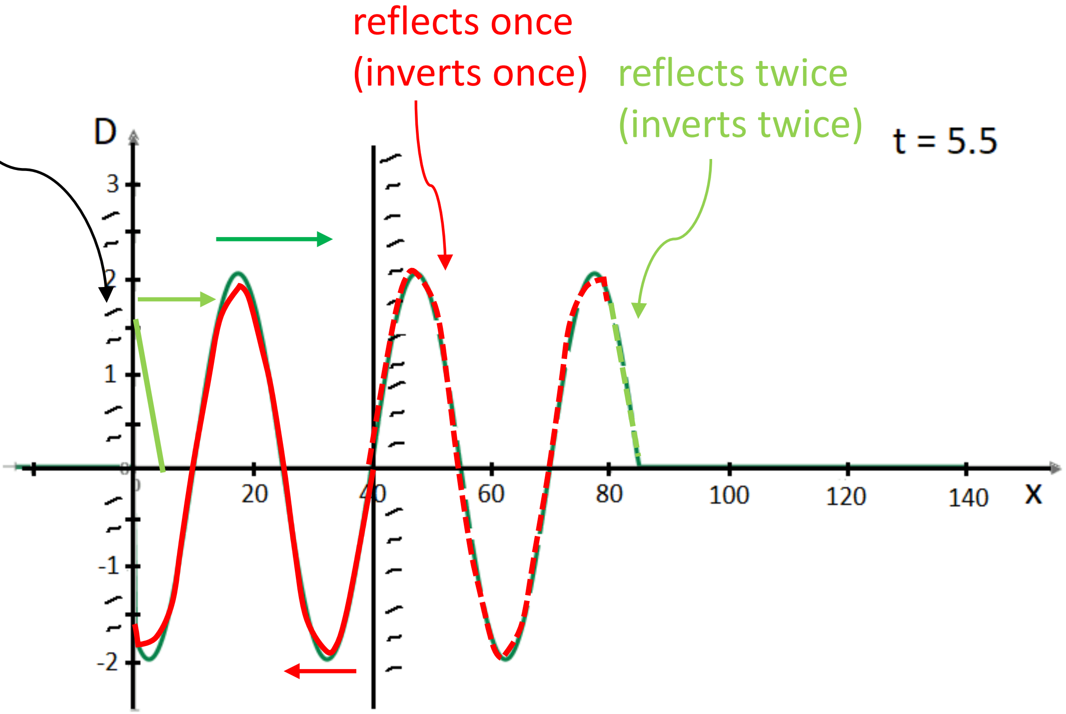




D.2 Single Source Interference: Standing Waves

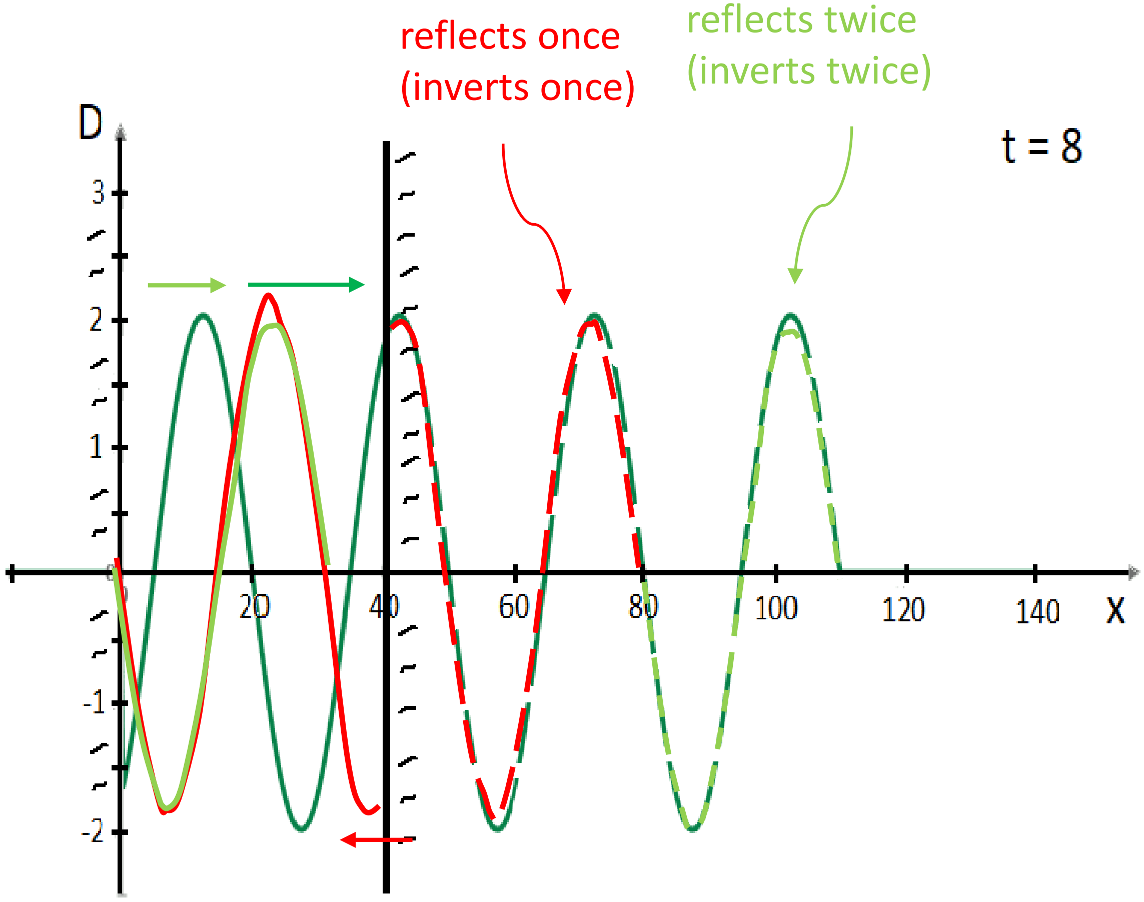
already going bad, because
this rightward going wave
is not overlapping the
original incident wave.



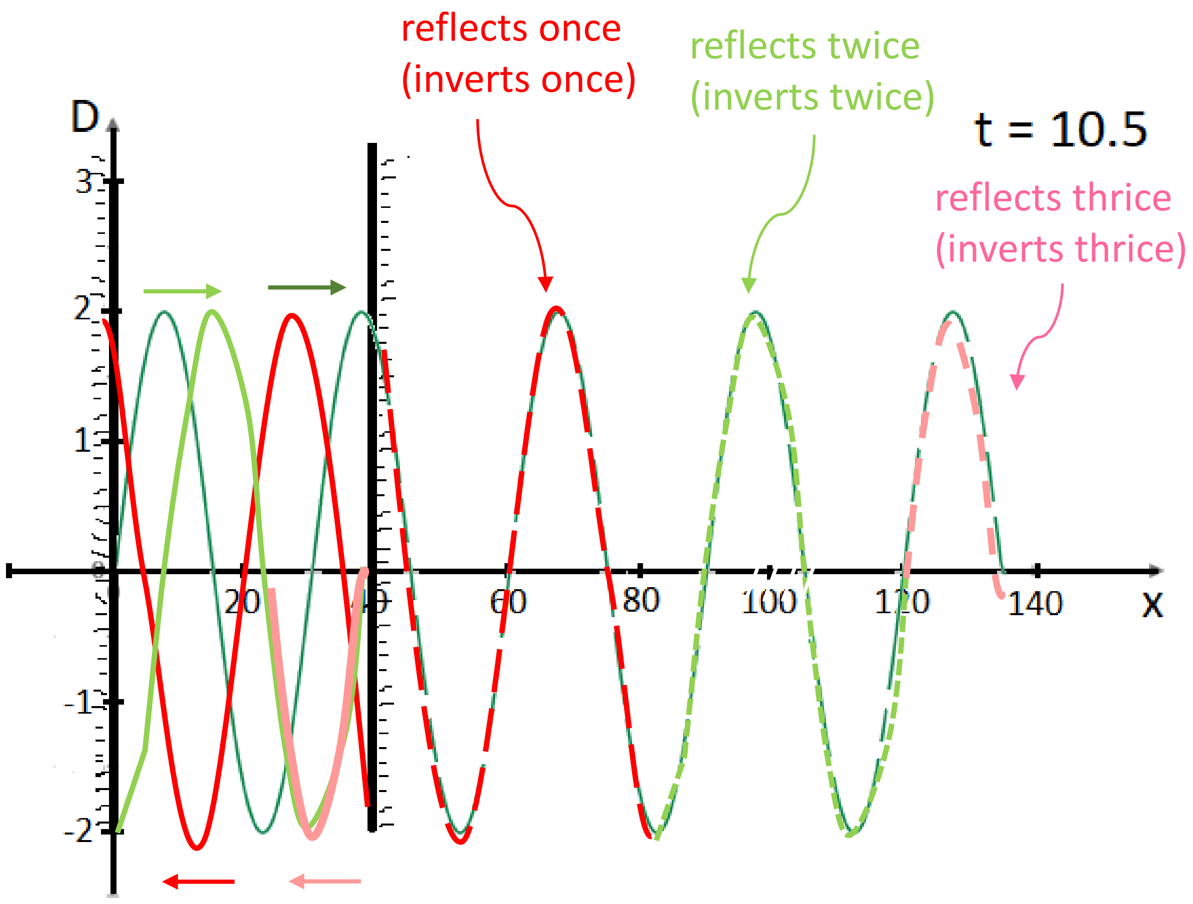


D.2 Single Source Interference: Standing Waves

Still bad

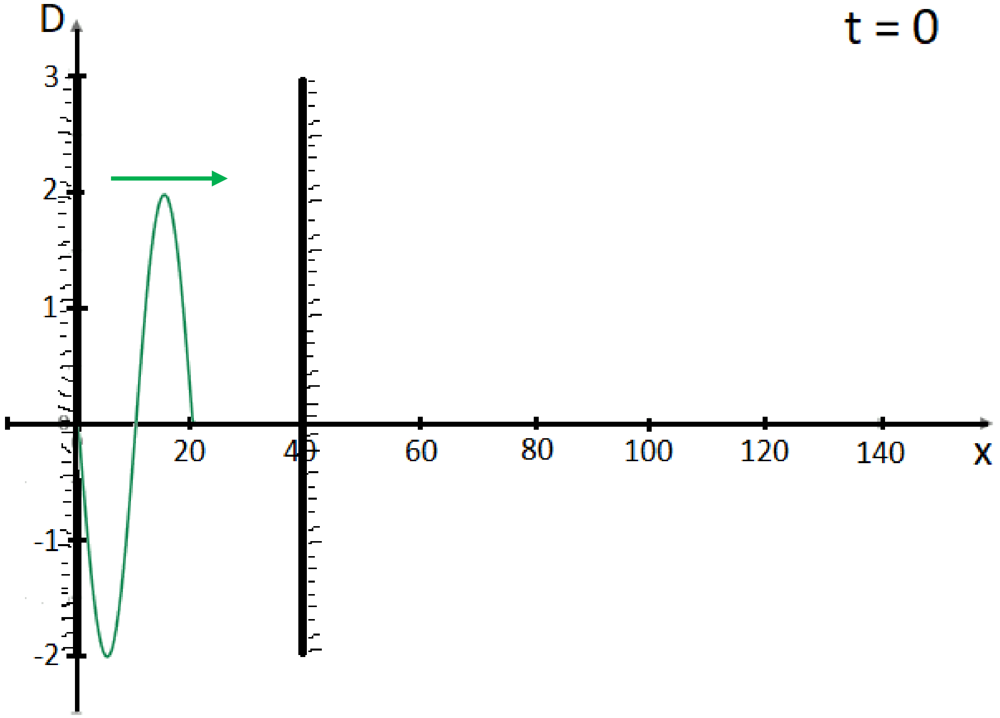


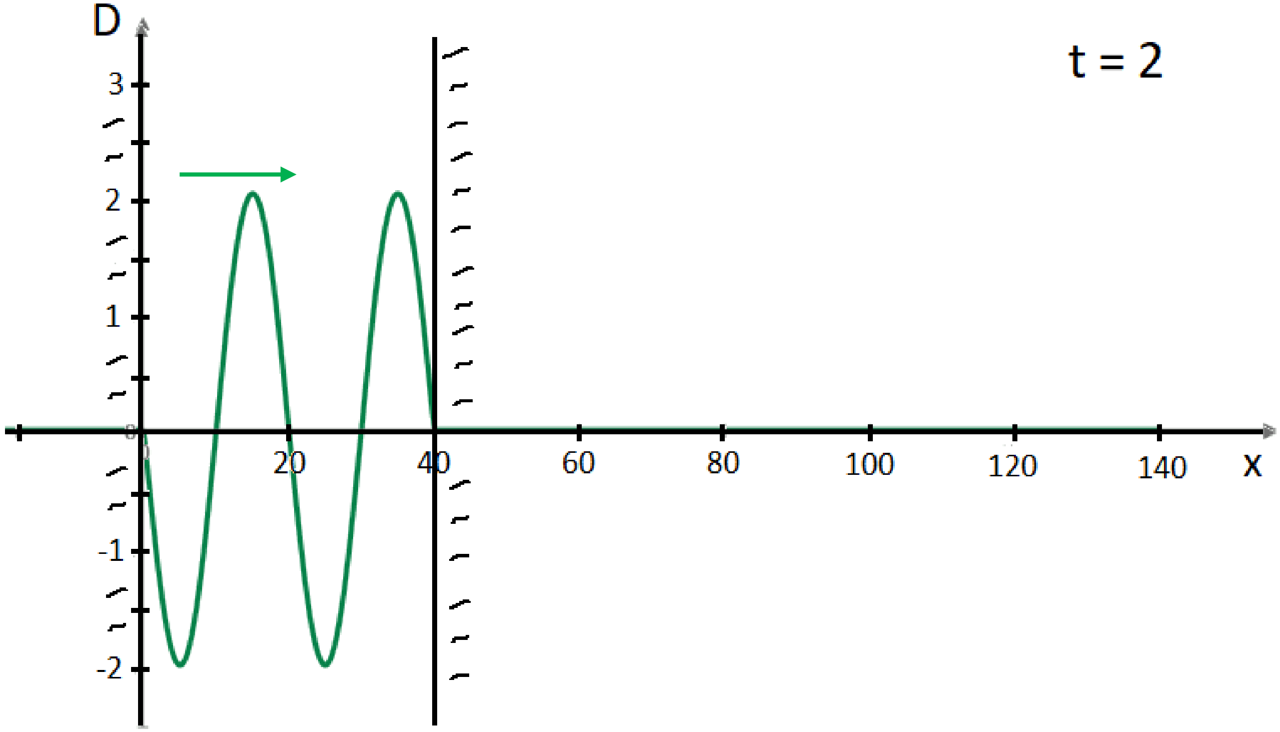
Yep, still bad



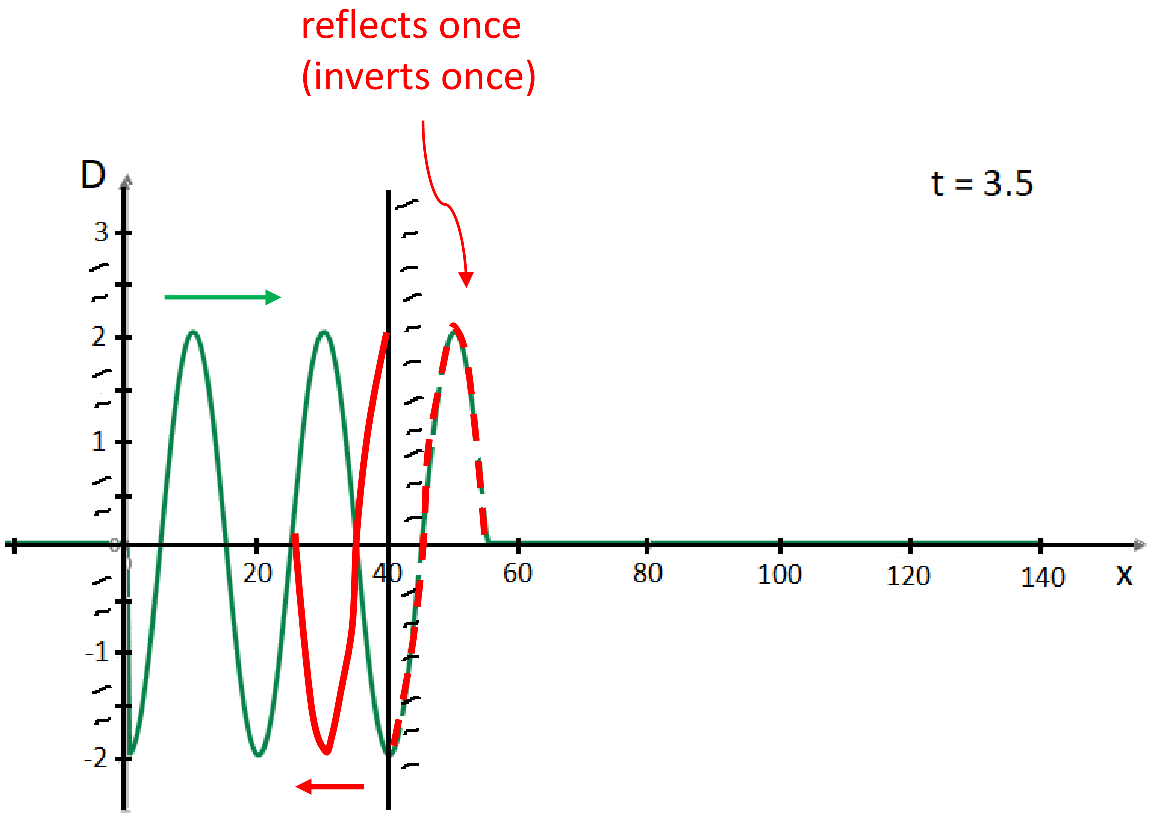
D.2 Single Source Interference: Standing Waves

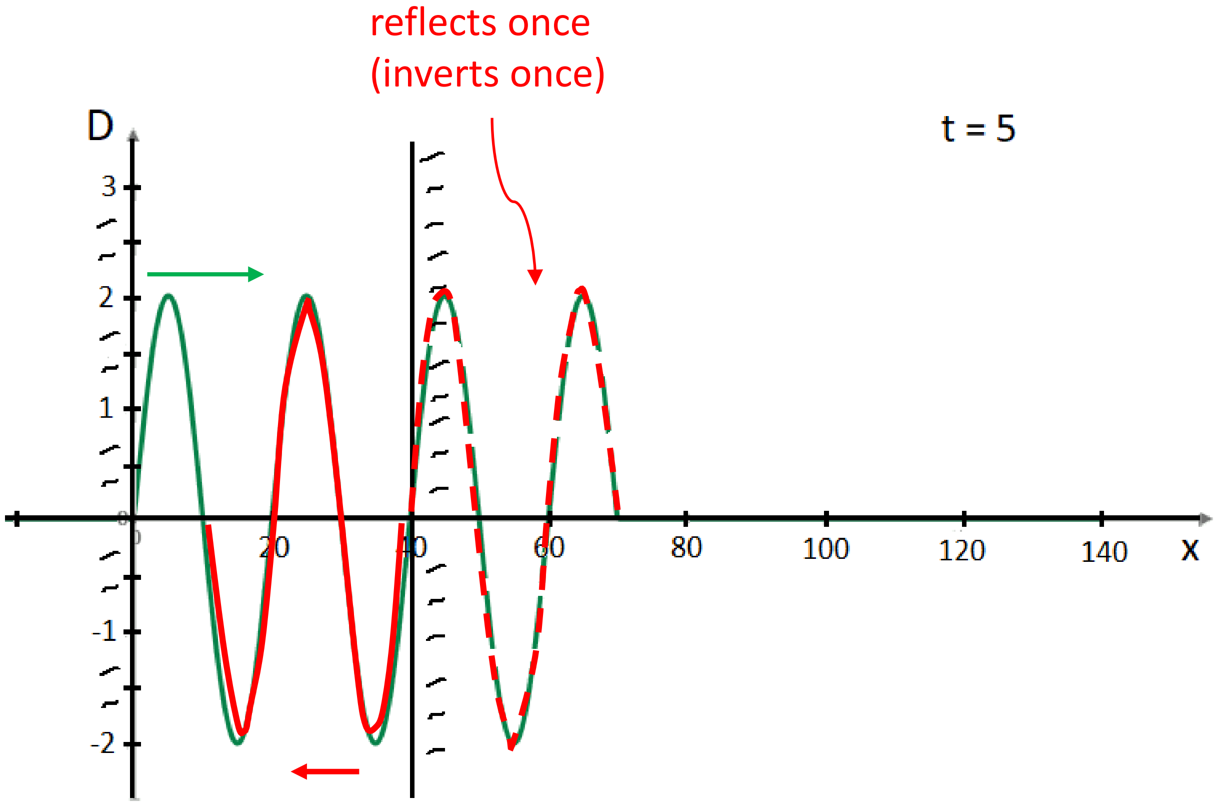
But there are cases where we do get resonance. Consider a λ = 20cm wave, traveling at 10cm/s as well.





D.2 Single Source Interference: Standing Waves



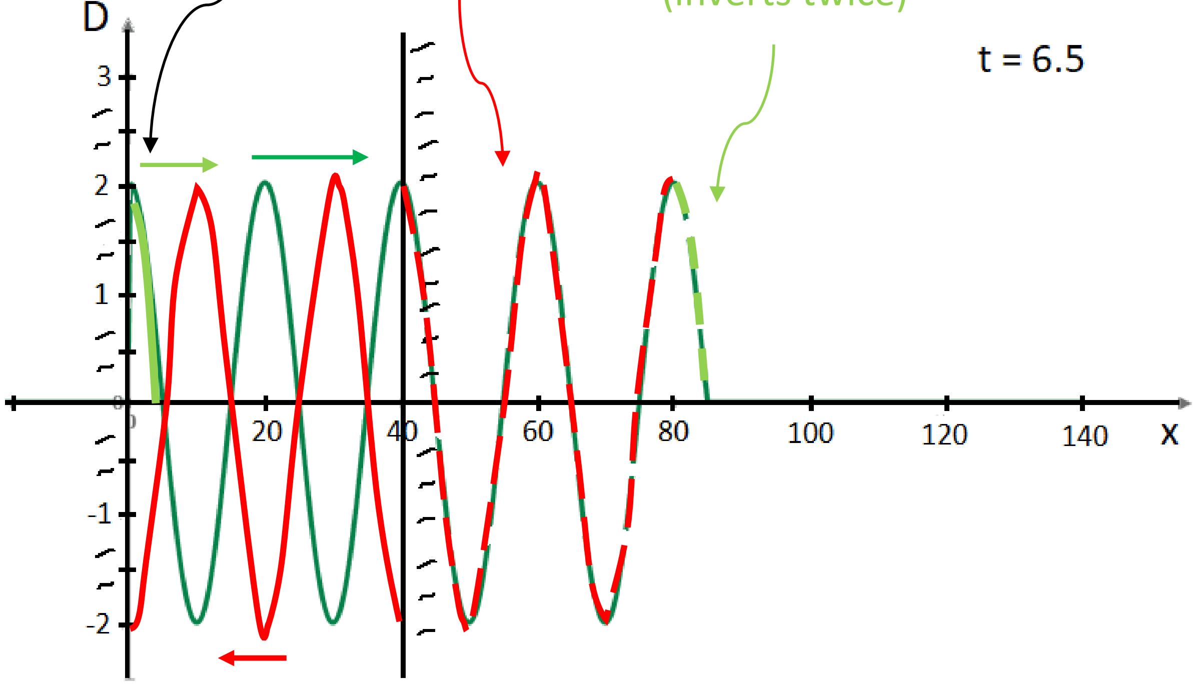


D.2 Single Source Interference: Standing Waves

starting off well, because the two rightward waves (greenish) overlap

reflects once (inverts once)

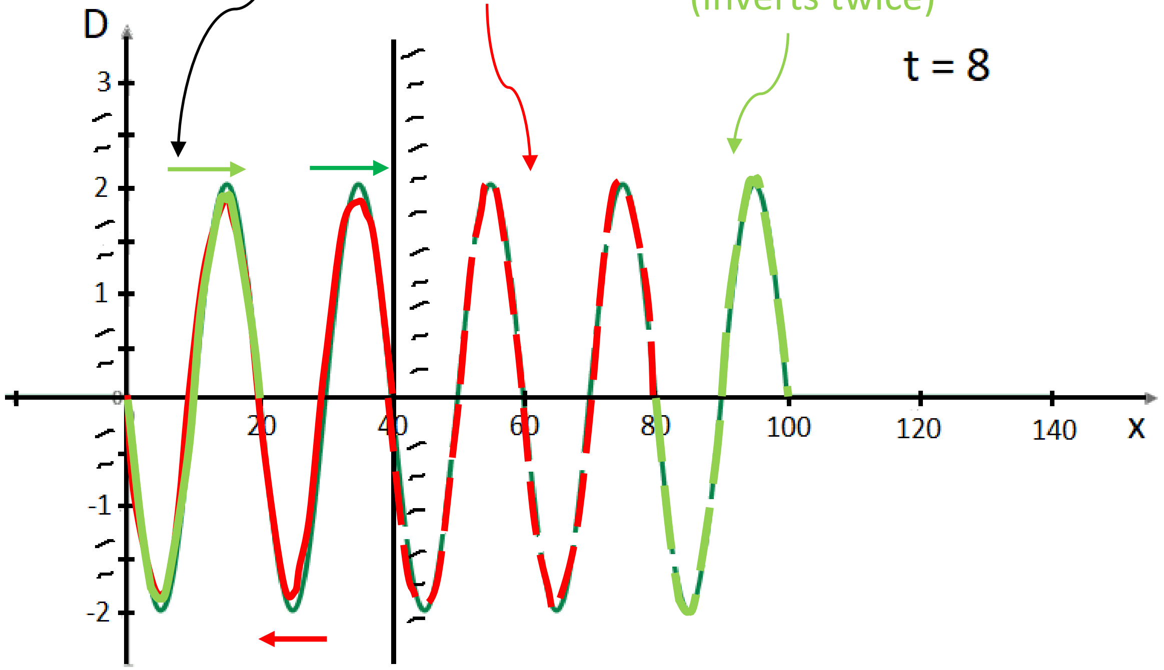
reflects twice (inverts twice)



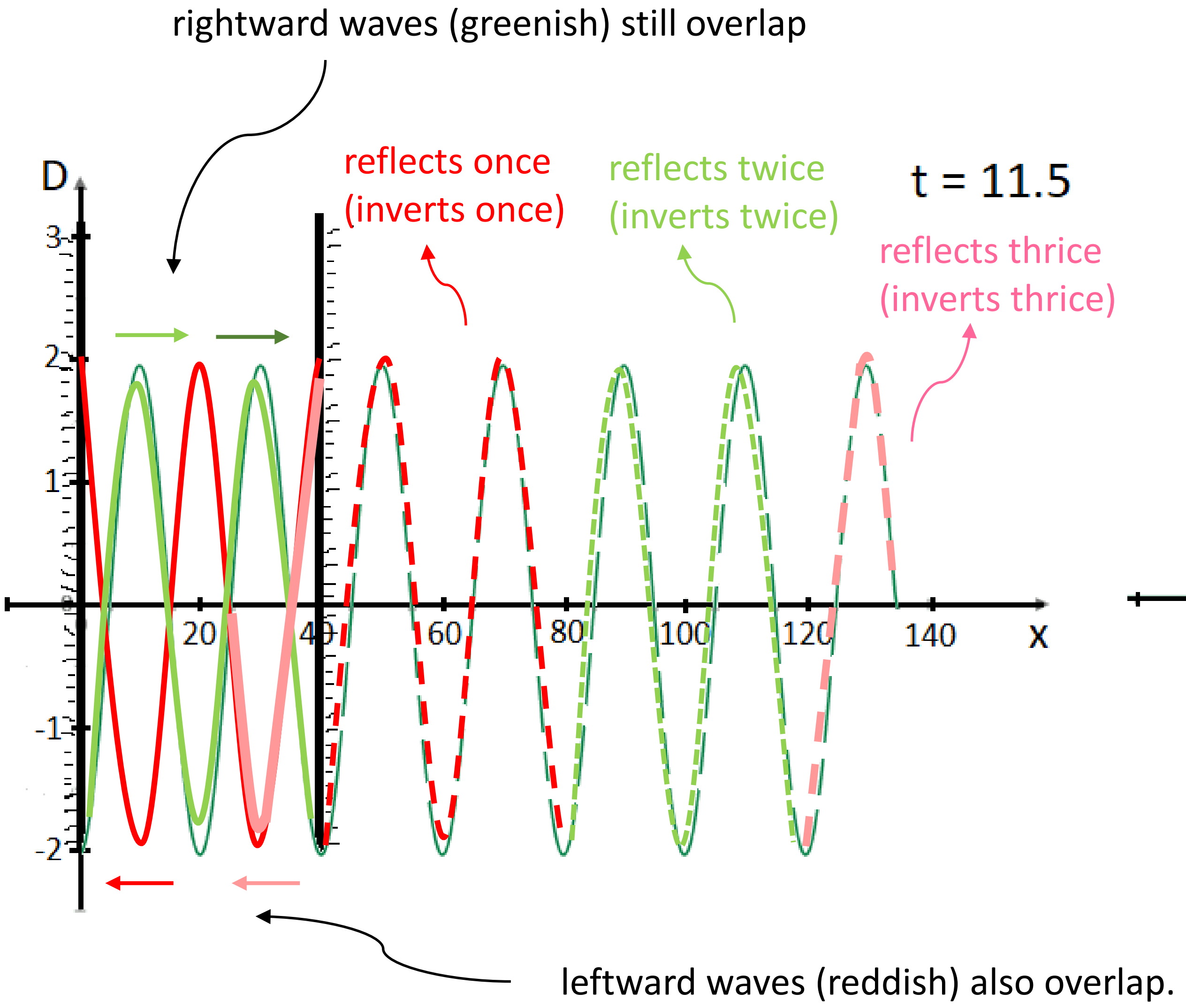
still going well because rightward (greenish) waves continue to overlap.

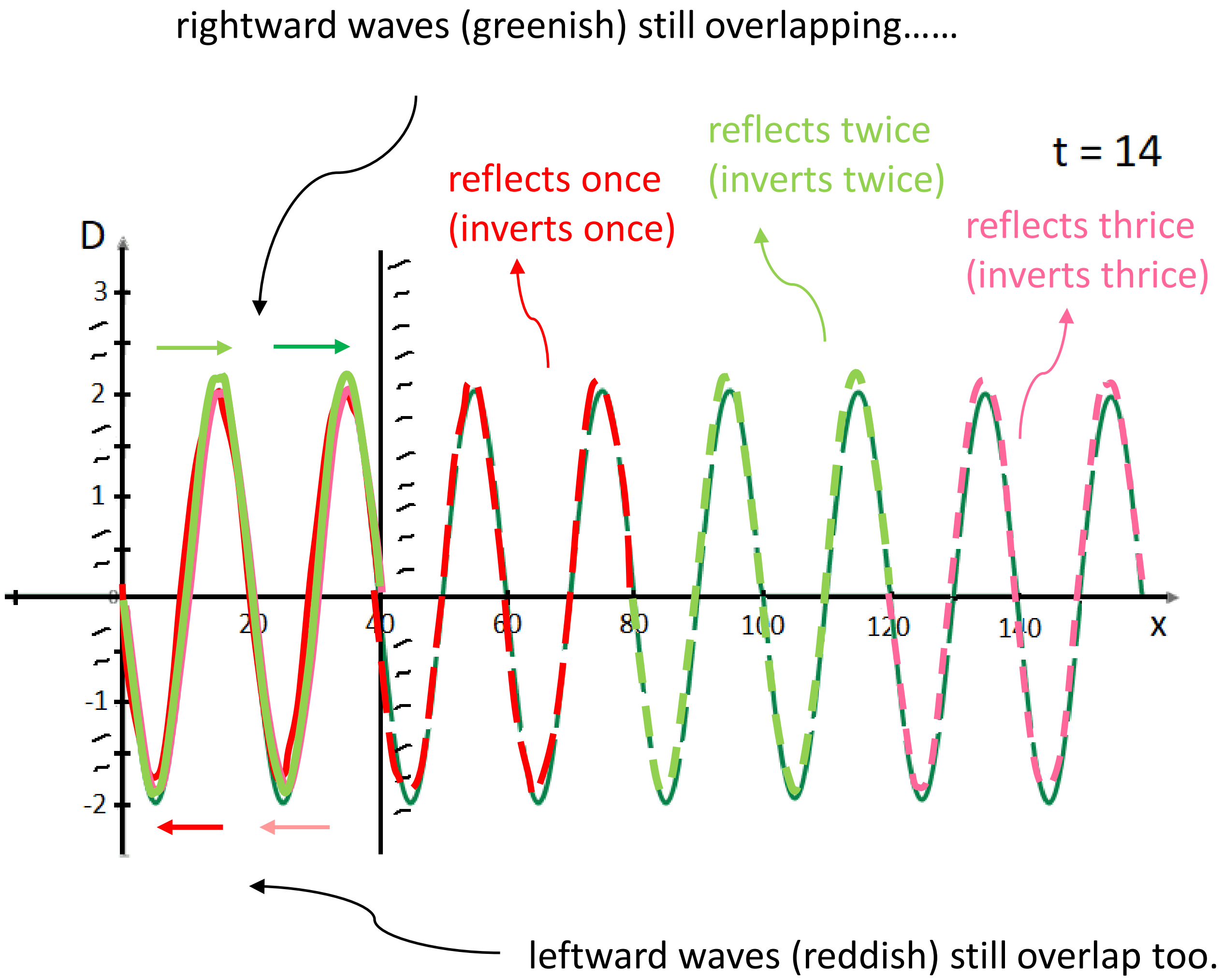
reflects once (inverts once)

reflects twice (inverts twice)



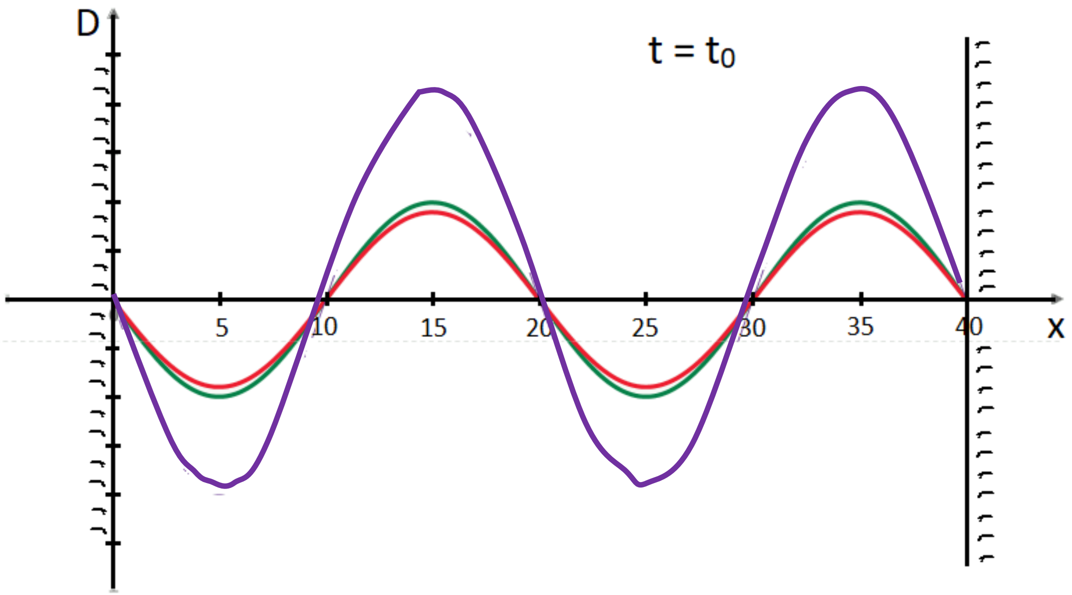
D.2 Single Source Interference: Standing Waves



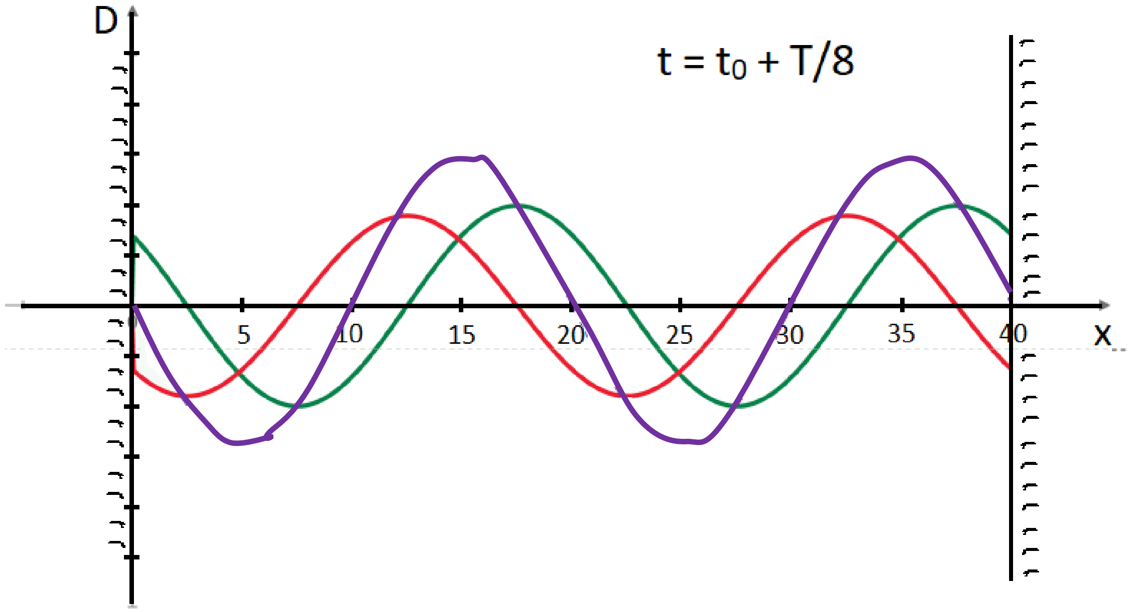


D.2 Single Source Interference: Standing Waves

Next question is, what will the string ultimately look like when the left/right waves are resonating? Well, we'll get interference between a super large rightward wave and a super large leftward wave. Let's see what *this* will look like. We'll revisit the previous case of λ = 20cm waves traveling 10cm/s, and note their period will be T = λ/v = 20/10 = 2s. So let's jump forward to some time, t_0, perhaps t_0 = 14, when the waves assumed this position.

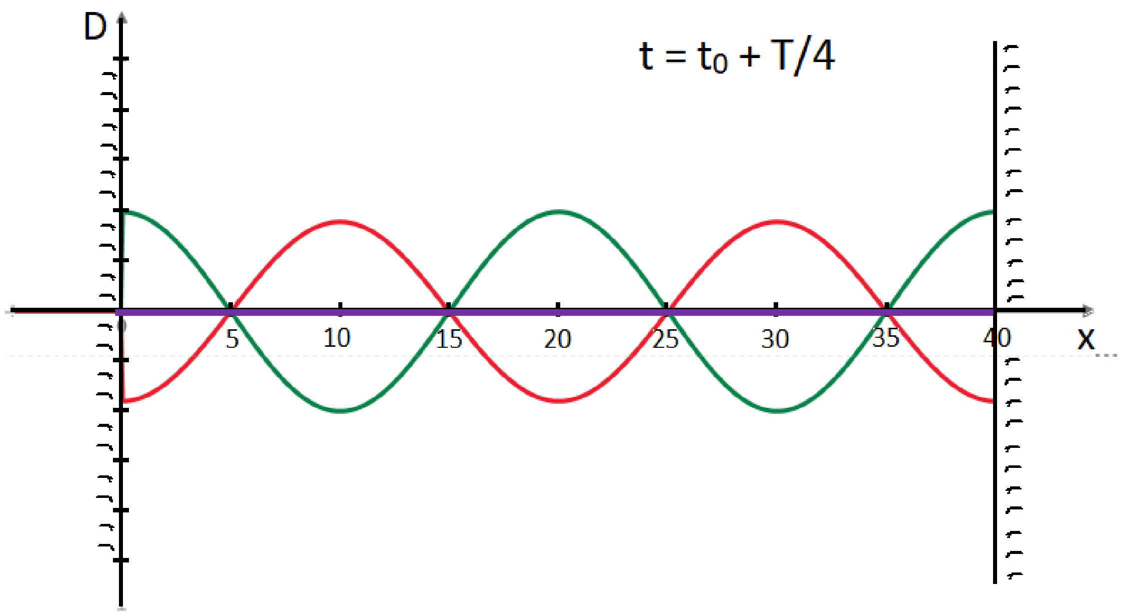


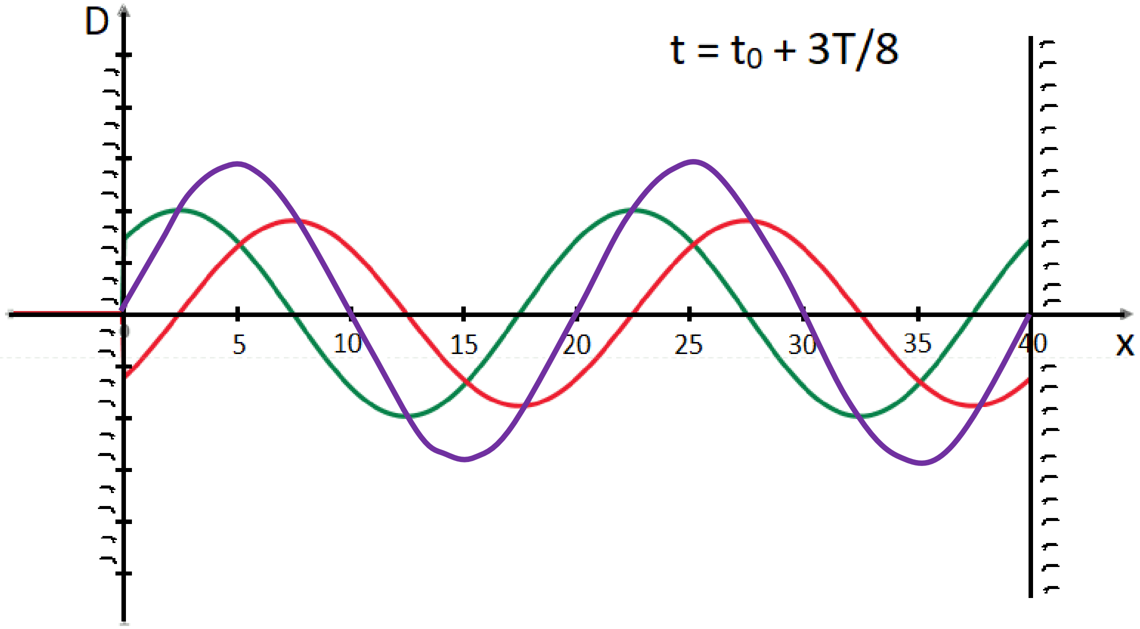
Then their superposition (purple) will be a double amplitude wave.



An eighth of a period (0.25s) later, the green waves will have traveled 2.5cm to right and the red waves 2.5cm to left. This will put them slightly out of phase, and the superposition will diminish. But note the *form* of the superposition doesn't change – it just *shrinks*.

D.2 Single Source Interference: Standing Waves

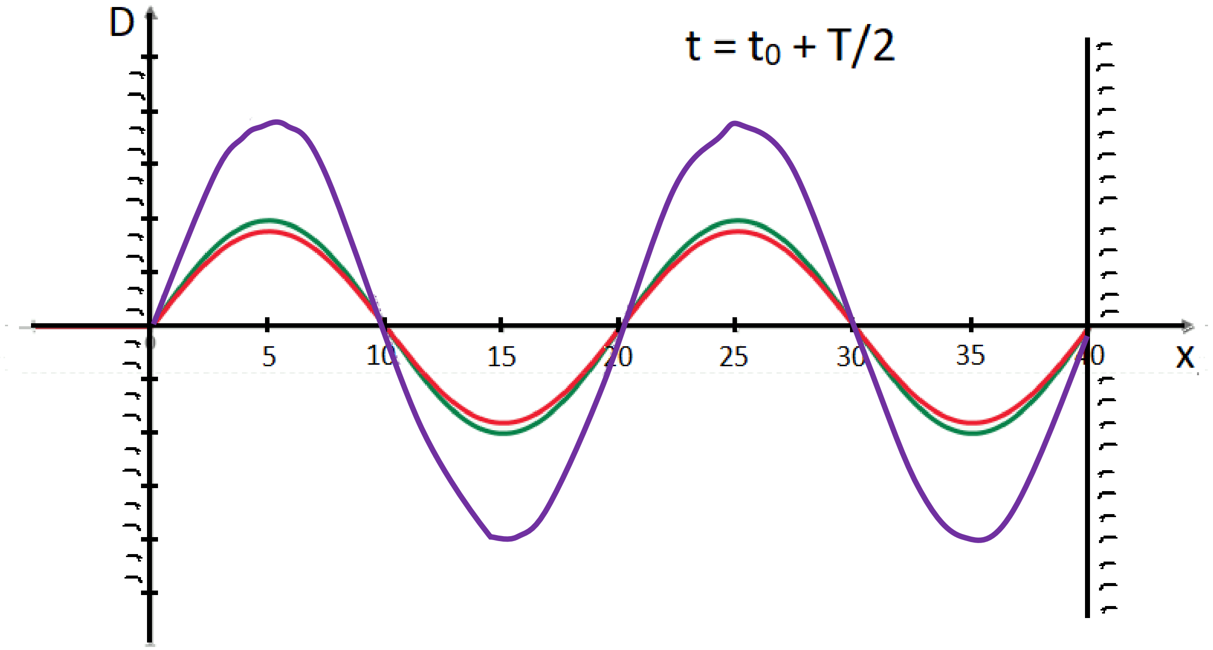


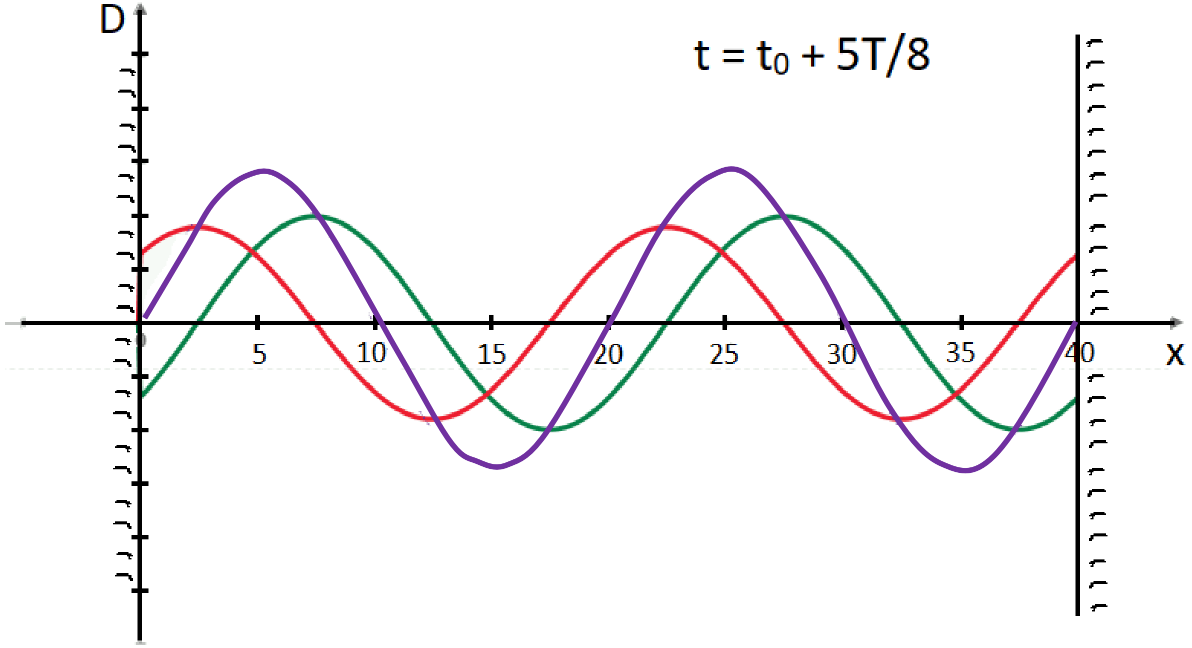


Another eighth of a period later, the green waves will have traveled another 2.5cm to right and the red waves 2.5cm to left. This will put them *completely* out of phase, and the superposition will be zero.

Another eighth of a period later, the green waves will have traveled another 2.5cm to right and the red waves 2.5cm to left. This will put them slightly out of phase, and the superposition will give a shape that looks like the $t = t_0 + T/8$ shape, except upside down.

D.2 Single Source Interference: Standing Waves

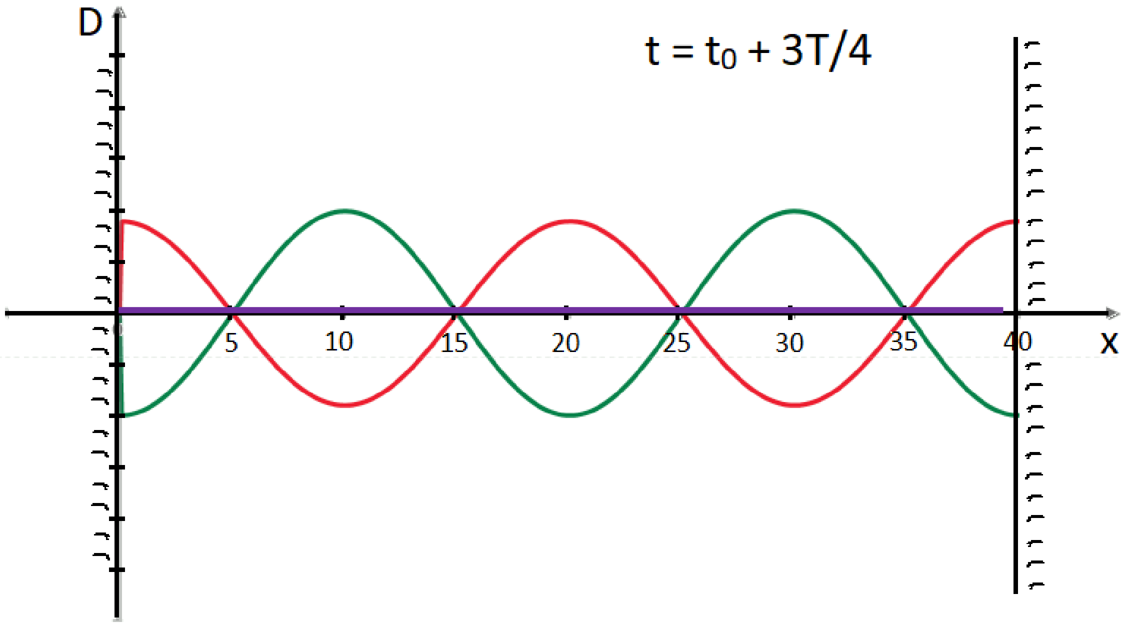


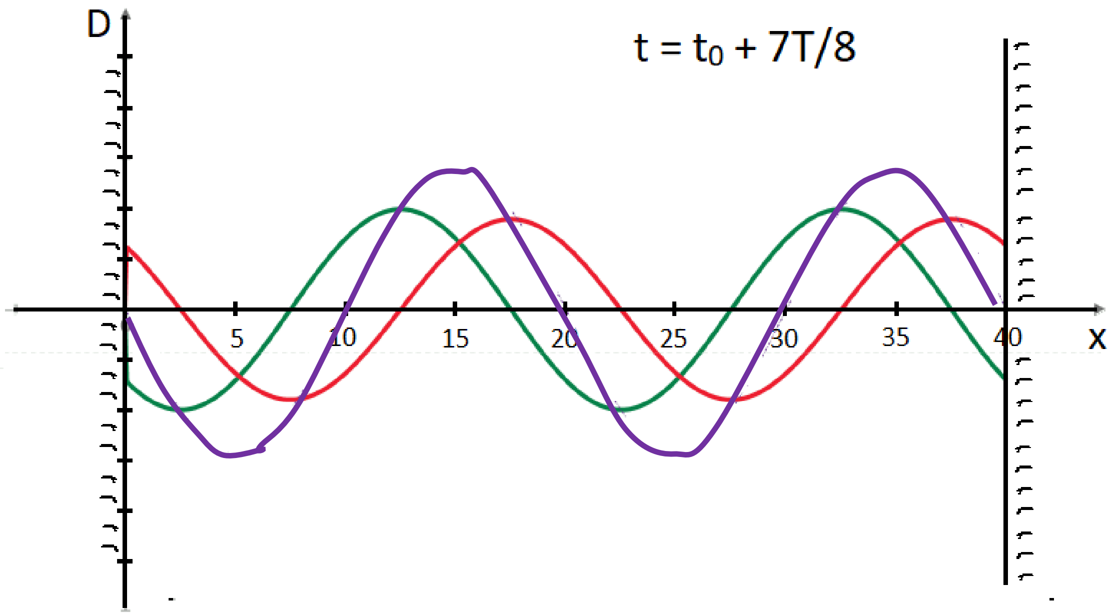


Another eighth of a period later, the green waves will have traveled another 2.5cm to right and the red waves 2.5cm to left. This will put them slightly completely in phase, and the superposition will give a shape that looks like the t_0 shape, except upside down.

Another eighth of a period later, the green waves will have traveled another 2.5cm to right and the red waves 2.5cm to left. This will put them slightly in phase, and the superposition will diminish again.

D.2 Single Source Interference: Standing Waves

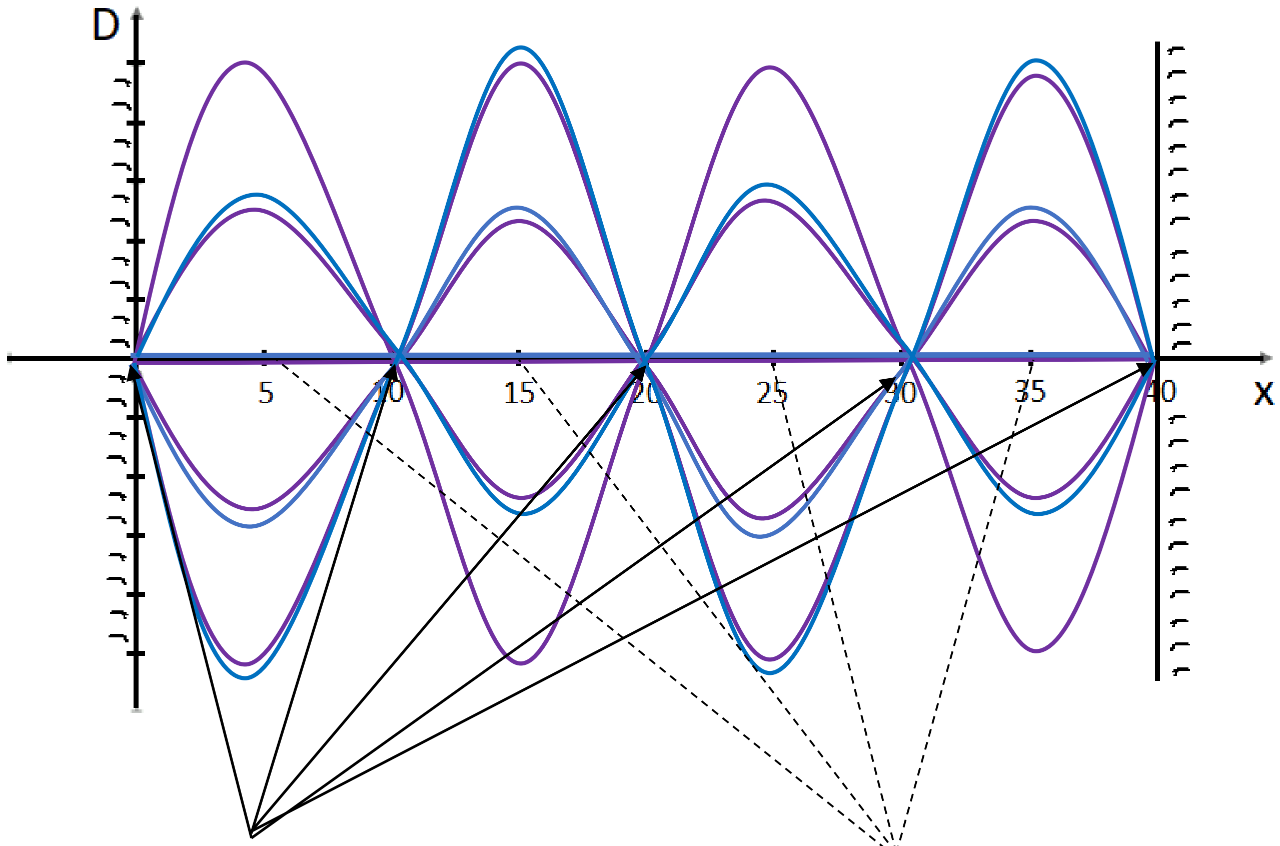




Another eighth of a period later, the green waves will have traveled another 2.5cm to right and the red waves 2.5cm to left. This will put them completely out of phase, and the superposition will be zero.

Another eighth of a period later, the green waves will have traveled another 2.5cm to right and the red waves 2.5cm to left. This will put them slightly in phase. And then at $t = t_0 + T$, we will return to the $t = t_0$ situation, and the pattern will continue repeat itself.

D.2 Single Source Interference: Standing Waves



All at once, just drawing the superpositions, we get the following set of shapes which will repeat themselves every period T.

Basic features:

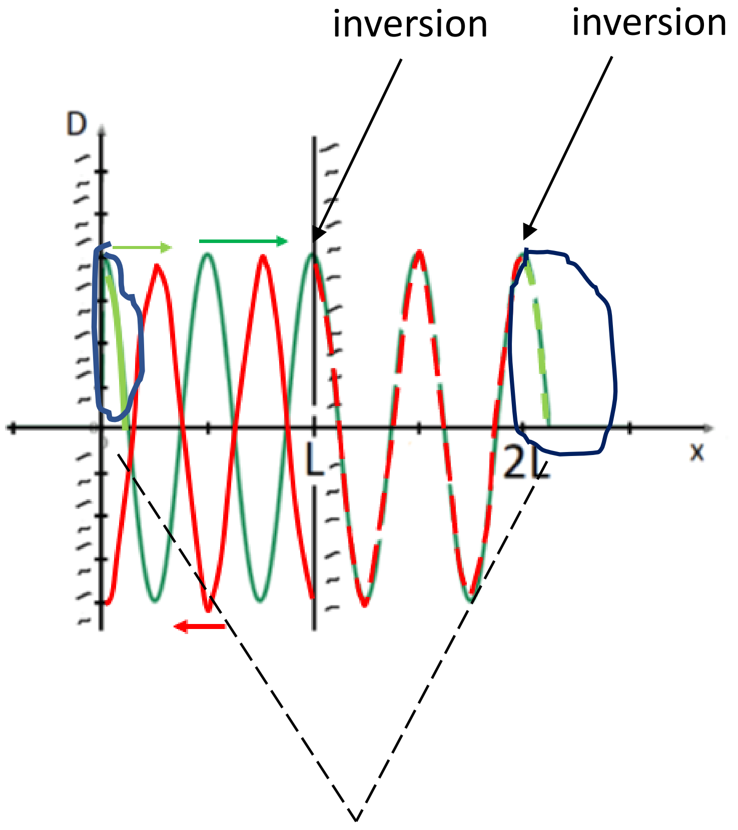
1. Same as shape of original wave.
2. Positioned so *nodes* are at any *hard* boundary (both), and *antinodes* at any *soft* boundary (none).
3. As time proceeds, wave *maintains* basic shape, with a *highly* augmented amplitude, but its size alternately shrinks and grows with time with a period T equal to the original wave's own period. This superposition is called a standing wave.

We should note points that do not move all. These are called ***nodes***. Note in particular the presence of nodes at the boundaries. There is always a node at a hard boundary.

And we should note points which oscillate with the largest amplitude. these are called ***antinodes***.

D.2 Single Source Interference: Standing Waves

So that's one particular example of resonance. But say we have a guitar string of a certain length, L. Which wavelengths in general will resonate? We began the section with the criteria for resonance, and if you go back through the illustration, or just think about it a little bit, you'll see that we'll get resonance automatically, if the 2nd reflected wave is in phase with the original incident wave. So we need,



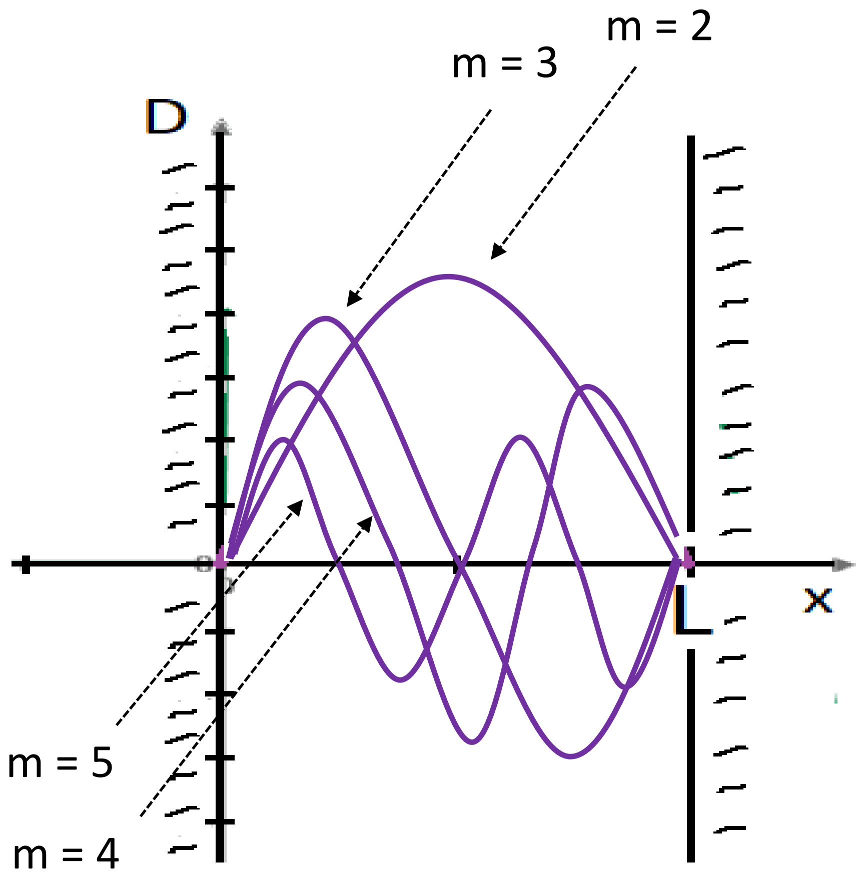
$$\Delta\varphi = 2\pi m \qquad m = 0, \pm 1, \pm 2, \text{etc.}$$

$$k\Delta x + I\pi = 2\pi m$$

$$\frac{2\pi}{\lambda}(2L) + 2\pi = 2\pi m$$

$$\lambda = \frac{2L}{m-1}$$

- only m ≥ 2 makes sense, is # of nodes
- m = 2 gives is 'fundamental', or 'first' harmonic
- m = 3, 4, 5, etc. are 2nd, 3rd, 4th, etc. harmonics



D.2 Single Source Interference: Standing Waves

Say we have our 40cm long guitar string. Its mass is m = 0.003kg. If we tighten it to tension T = 380N, what would be the first three resonant frequencies?

Repeating our analysis, the resonant waveforms are given by:

$$\Delta\varphi = 2\pi m$$

$$k\Delta x + I\pi = 2\pi m$$

$$\left(\frac{2\pi}{\lambda}\right)(2\times 40\text{cm}) + 2\cdot\pi = 2\pi m$$

$$\lambda = \frac{80\text{cm}}{m-1} \qquad m \geq 2$$

So the first three resonant wavelengths are:

$$\lambda = \frac{80\text{cm}}{(2,3,4)-1} = 80\text{cm}, 40\text{cm}, 27\text{cm}$$

then to get the resonant frequencies we just use:

$$f = \frac{v}{\lambda}$$

but what's v?

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{380}{(0.003/0.40)}} = 225\,\text{m/s}$$

therefore,

$$f = \frac{225}{0.80}, \frac{225}{0.40}, \frac{225}{0.27} = 281\text{Hz},\ 561\text{Hz},\ 833\text{Hz}$$

'fundamental' frequency

higher harmonics are multiples of fundamental

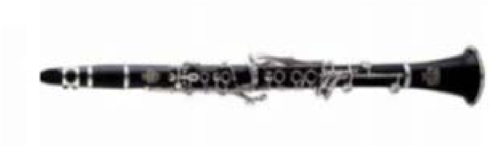
D.2 Single Source Interference: Standing Waves

Which frequency do you hear when you pluck the string?

You'll hear the one whose waveform most closely resembles the shape of the plucked guitar string. Typically, this will be the fundamental waveform. And so you'd expect to hear 281Hz.

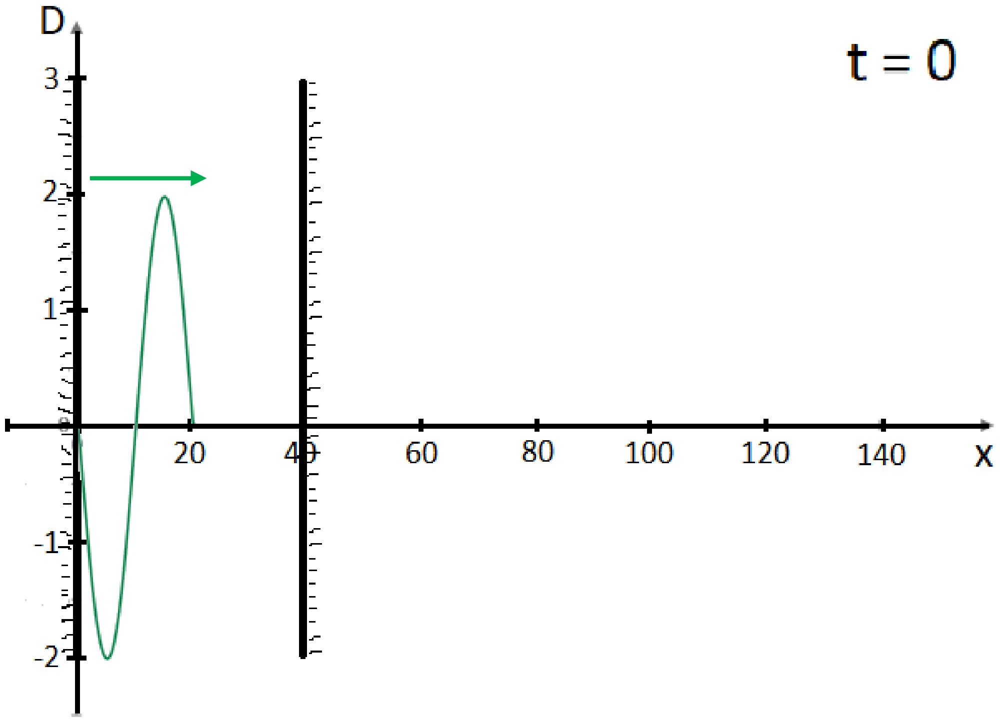
What would happen to these frequencies if you increased the tension in the string, or the mass of the string?

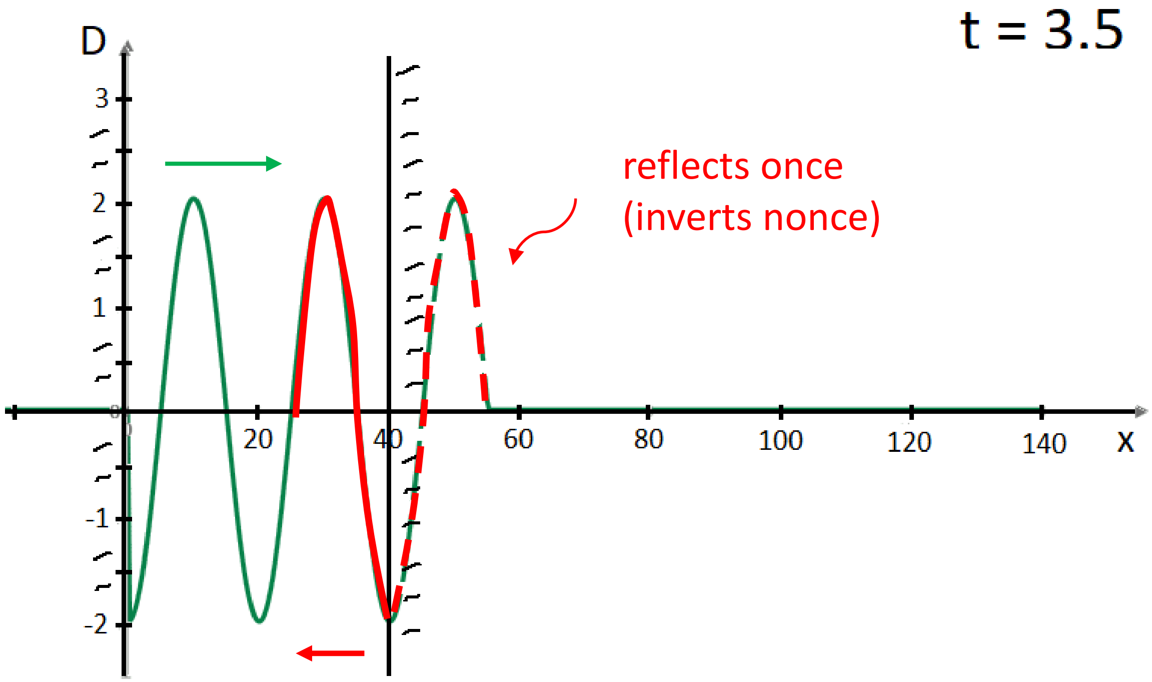
Increasing tension would increase v, and so increase f. Increasing m would decrease v, and so decrease f.

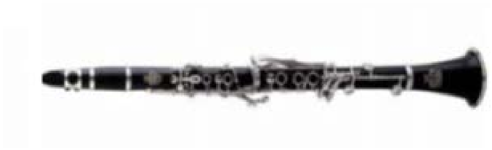


D.2 Single Source Interference: Standing Waves

Let's consider a different scenario – the clarinet. It would have a hard boundary at the mouthpiece, and a soft boundary at the end of the instrument (the bell?). That means when a wave reflects off the mouthpiece it will invert, but when it reflects off the bell, it will not invert. Let's consider the propagation of a 20cm wave doing 10cm/s down a 40cm long clarinet, and see if the wave will resonate or not. I'll speed up the times a little bit.



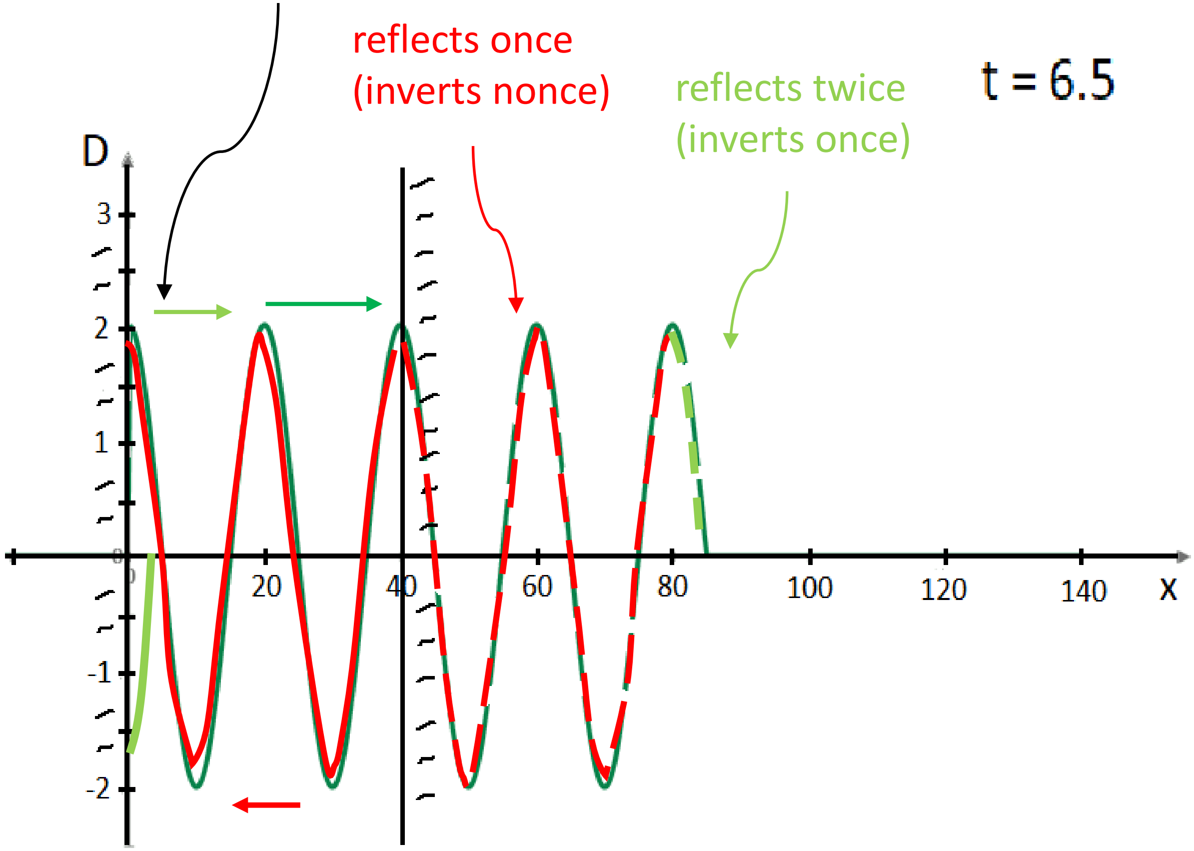


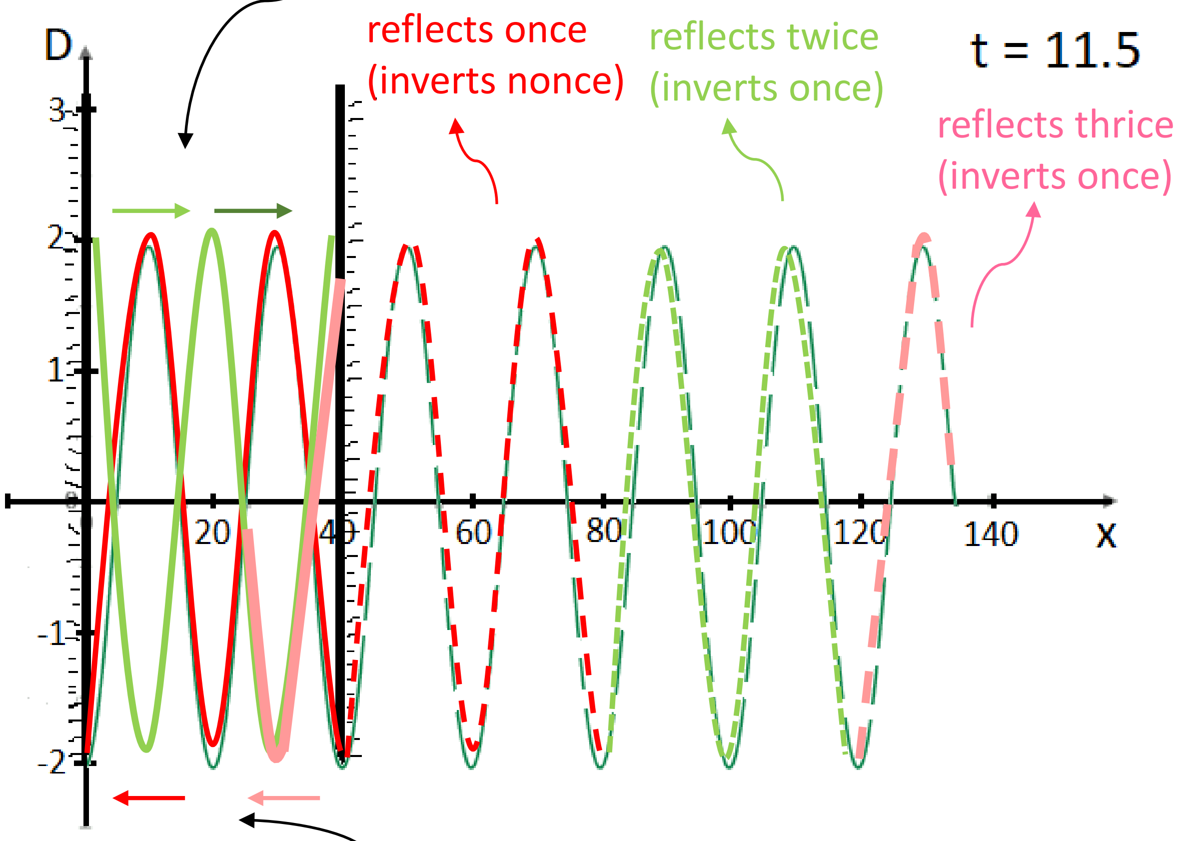


D.2 Single Source Interference: Standing Waves

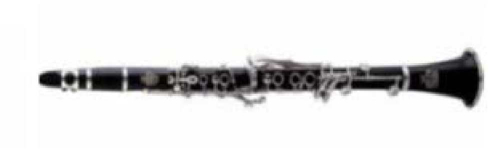
going bad because rightward (greenish) waves don't overlap.

rightward waves (greenish) ain't overlapping



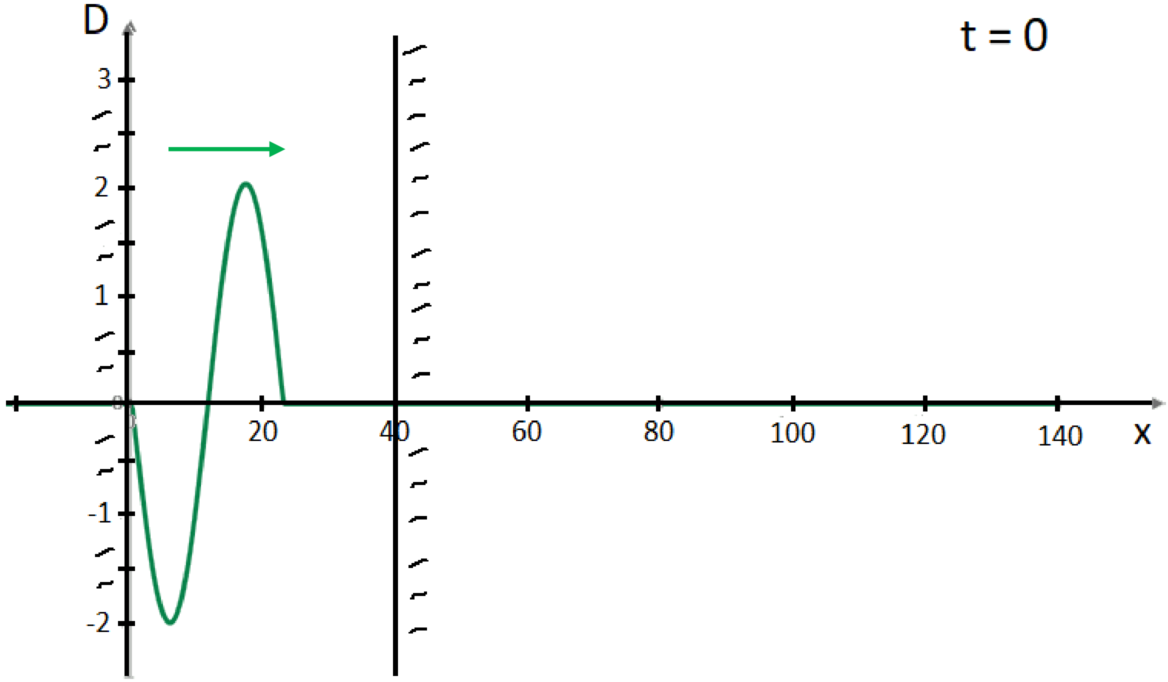


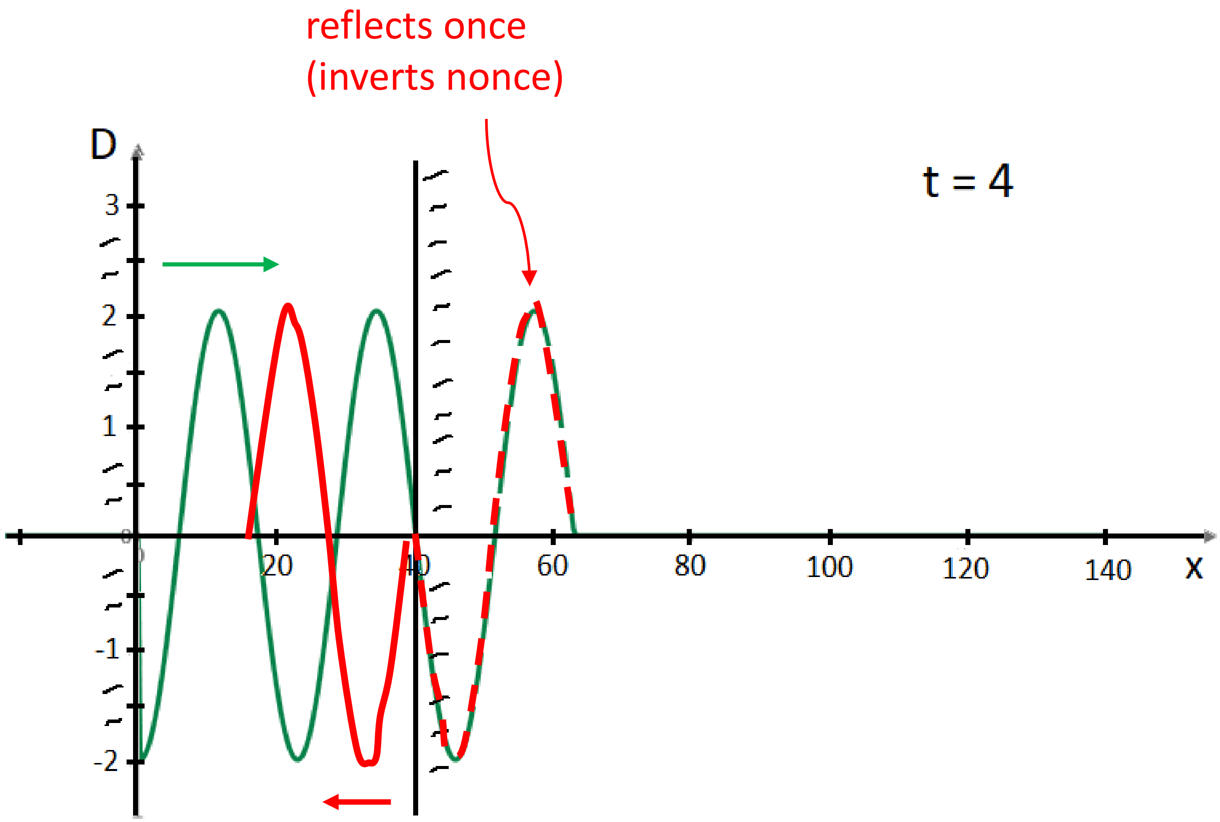
neither are leftward (reddish) waves.

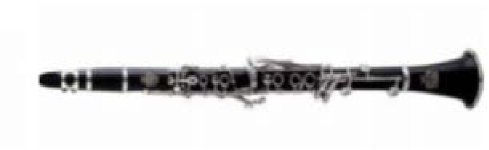


D.2 Single Source Interference: Standing Waves

So that wavelength doesn't resonate, but another one might. For instance, consider just a slightly larger wave: λ = 23cm, and doing 10cm/s again.





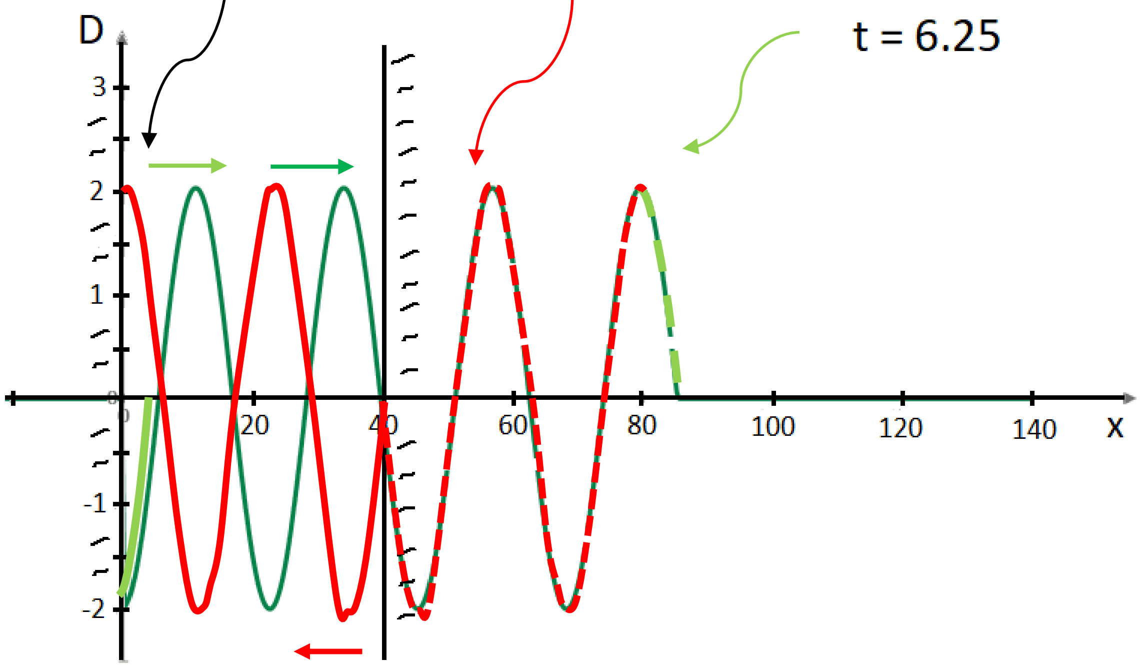


D.2 Single Source Interference: Standing Waves

auspicious: rightward (greenish) waves overlap.

reflects once (inverts nonce)

reflects twice (inverts once)

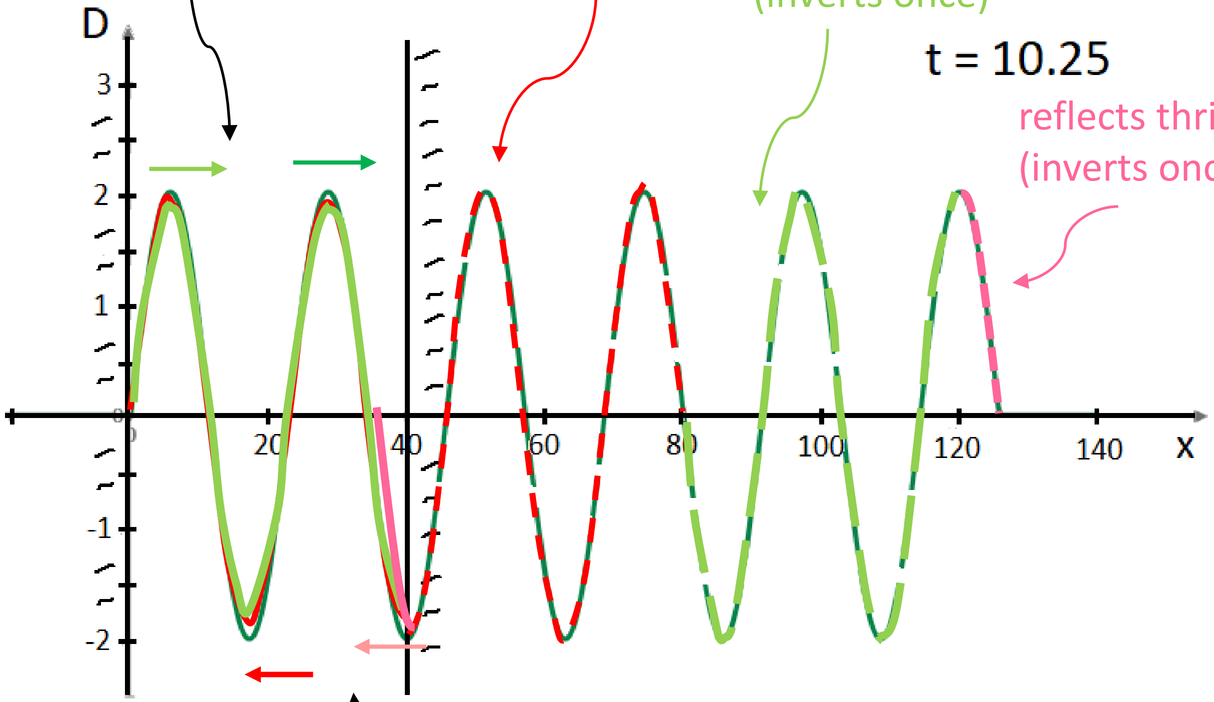


rightward waves (greenish) are still overlapping

reflects once (inverts nonce)

reflects twice (inverts once)

reflects thrice (inverts once)



And so are leftward (reddish) waves, so we seem to be in resonance.

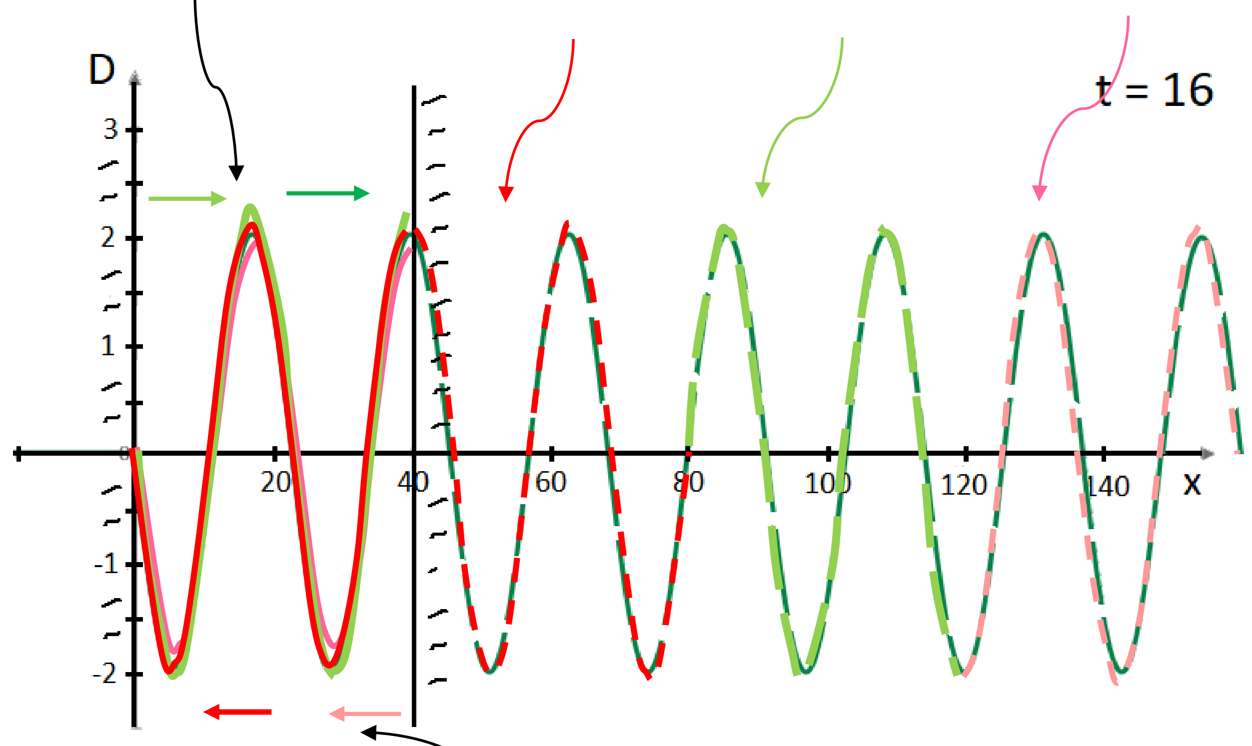
D.2 Single Source Interference: Standing Waves

rightward waves (greenish) are still overlapping

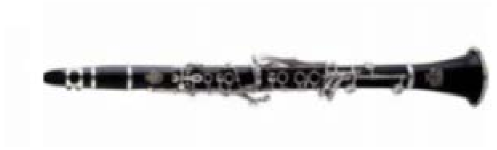
reflects once (inverts nonce)

reflects twice (inverts once)

reflects thrice (inverts once)

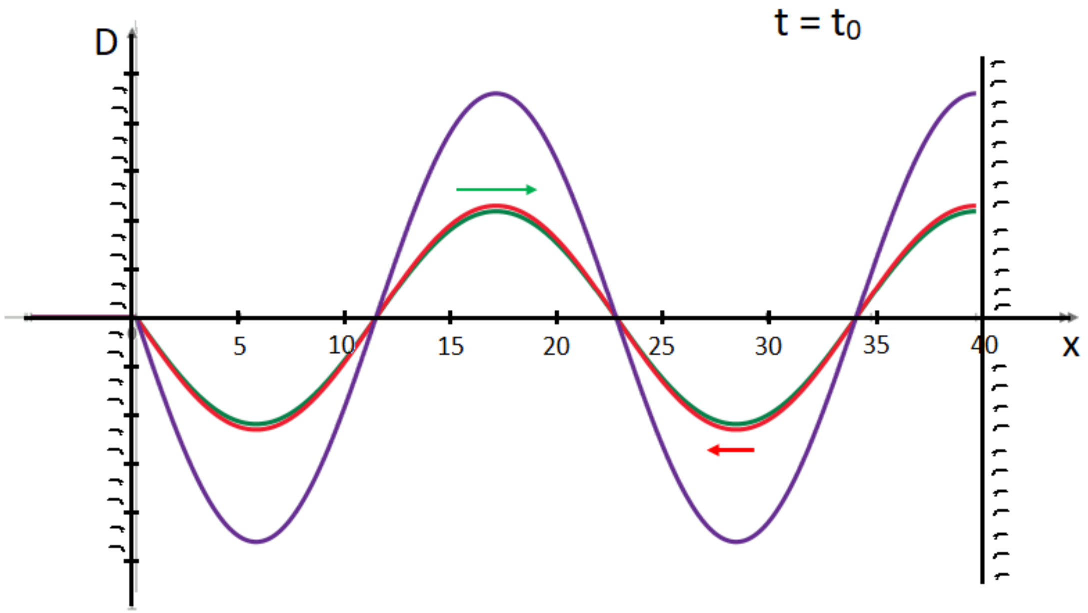


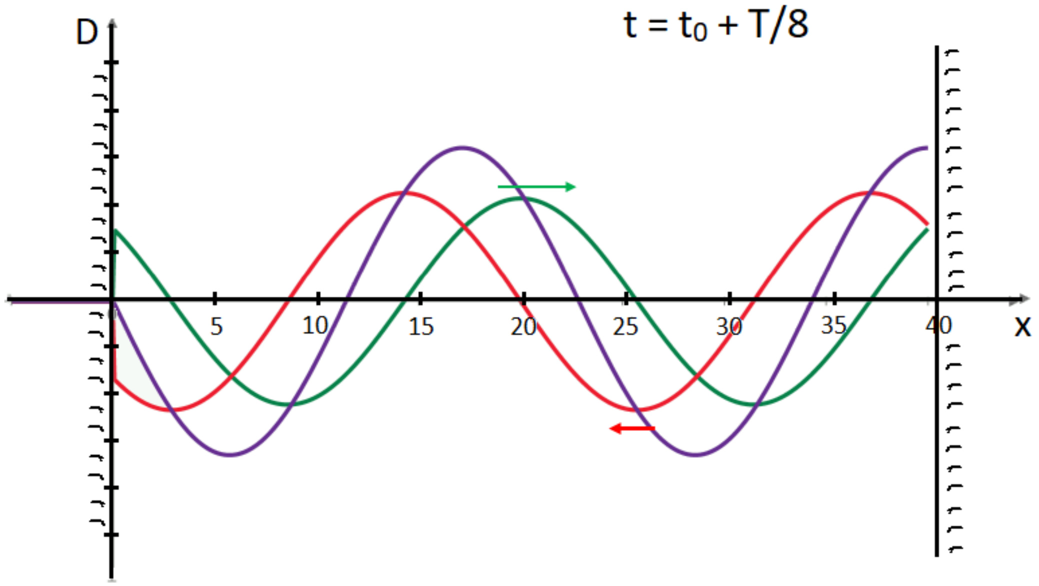
And so are leftward (reddish) waves, so we *are* resonating.

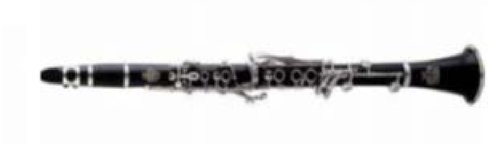


D.2 Single Source Interference: Standing Waves

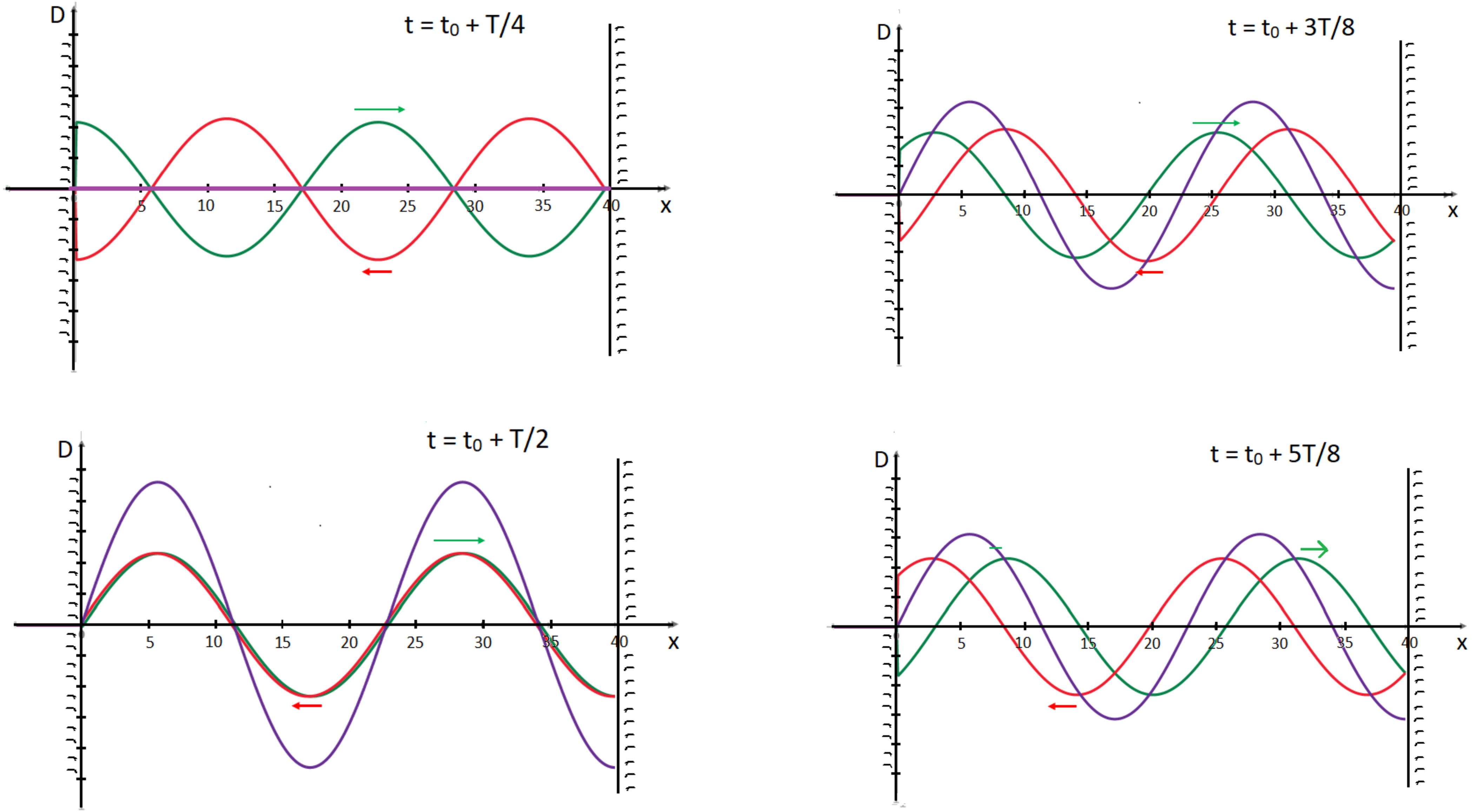
Now let's see what this resonant wave will look like on the string. Green is going right; red to the left, and purple is the superposition. I'll progress time in increments of an eight of period, starting from some time when the waves overlap, like they do at t = 14s.

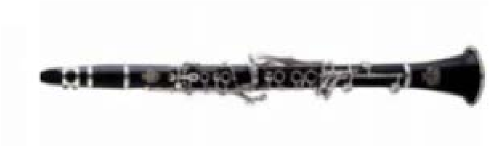




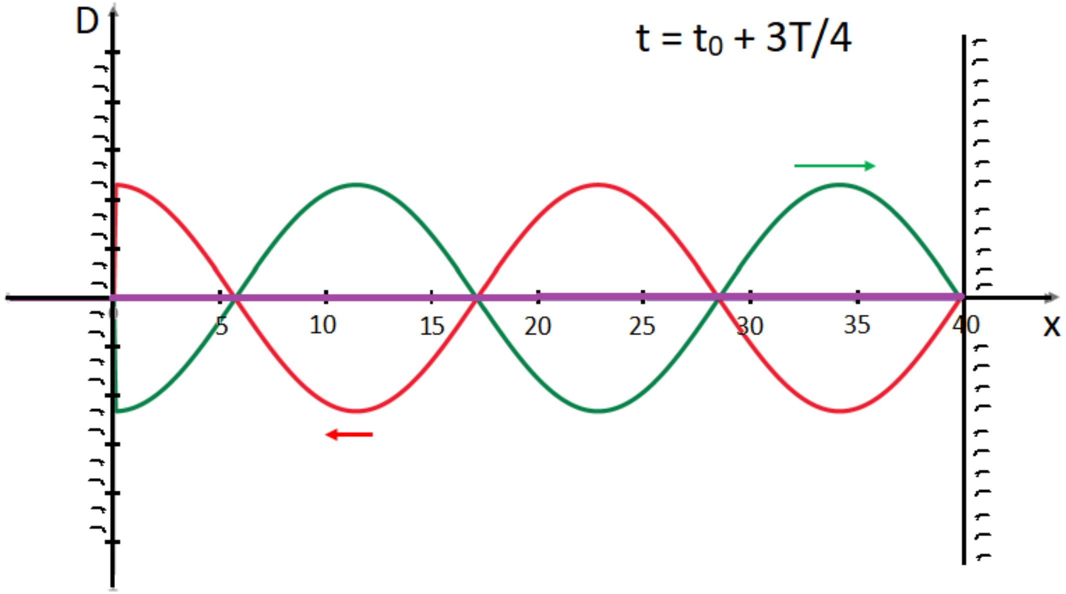


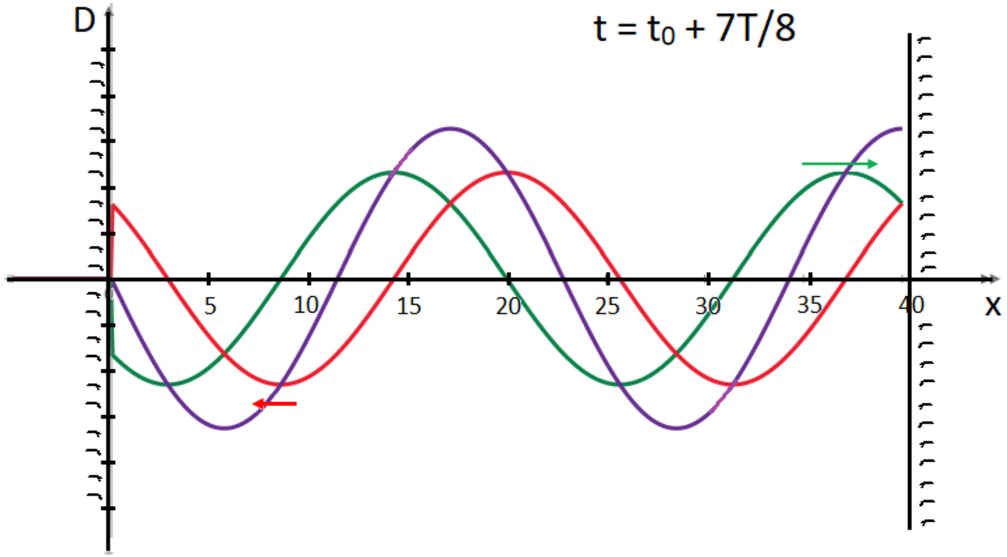
D.2 Single Source Interference: Standing Waves

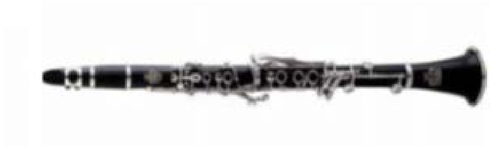




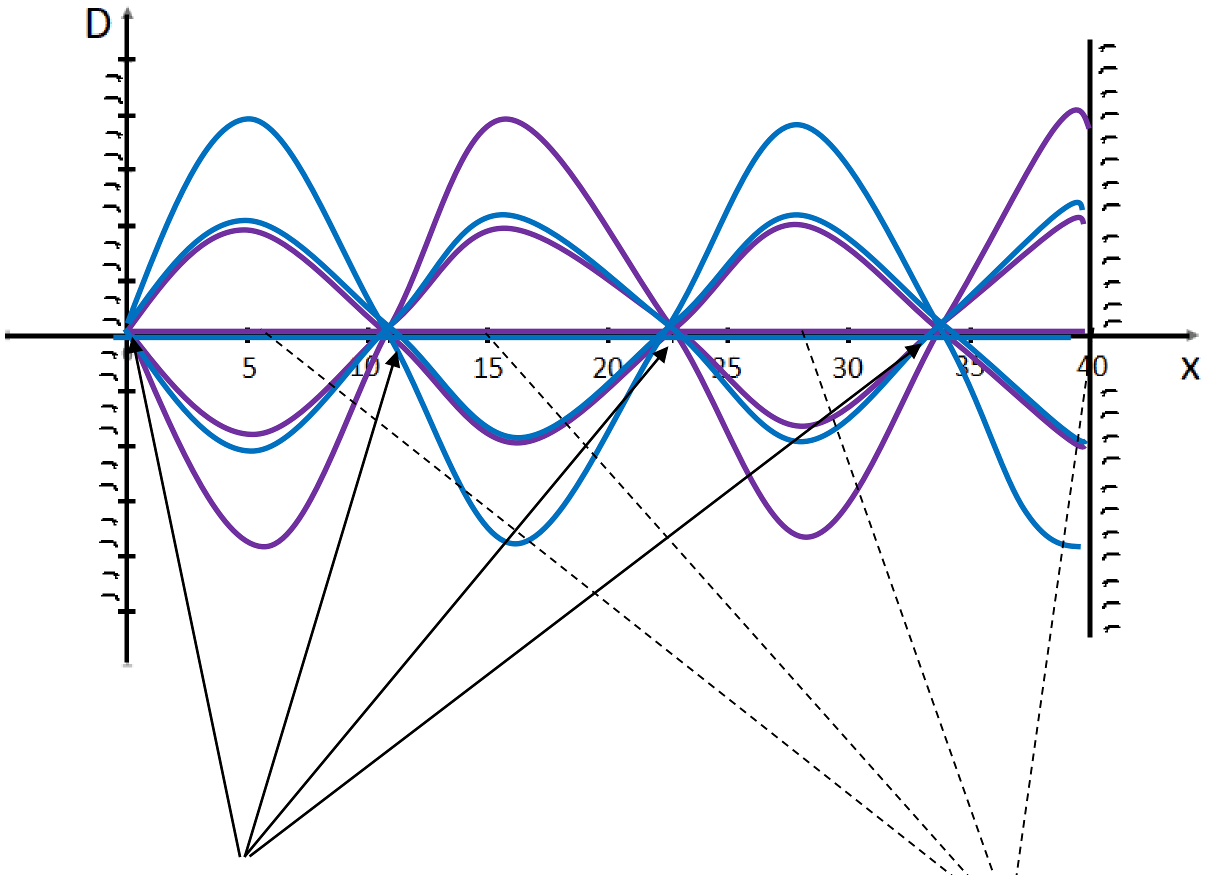
D.2 Single Source Interference: Standing Waves







D.2 Single Source Interference: Standing Waves



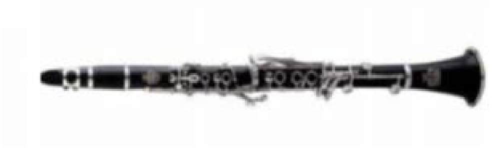
All at once, just drawing the superpositions, we get another standing wave which will repeat itself every period T.

Basic features again:

1. Same as shape of original wave.
2. Positioned so *nodes* are at any *hard* boundary (one), and *antinodes* at any *soft* boundary (one).
3. As time proceeds, wave *maintains* basic shape, with a *highly* augmented amplitude, but its size alternately shrinks and grows with time with a period T equal to the original wave's own period. This superposition is called a standing wave.

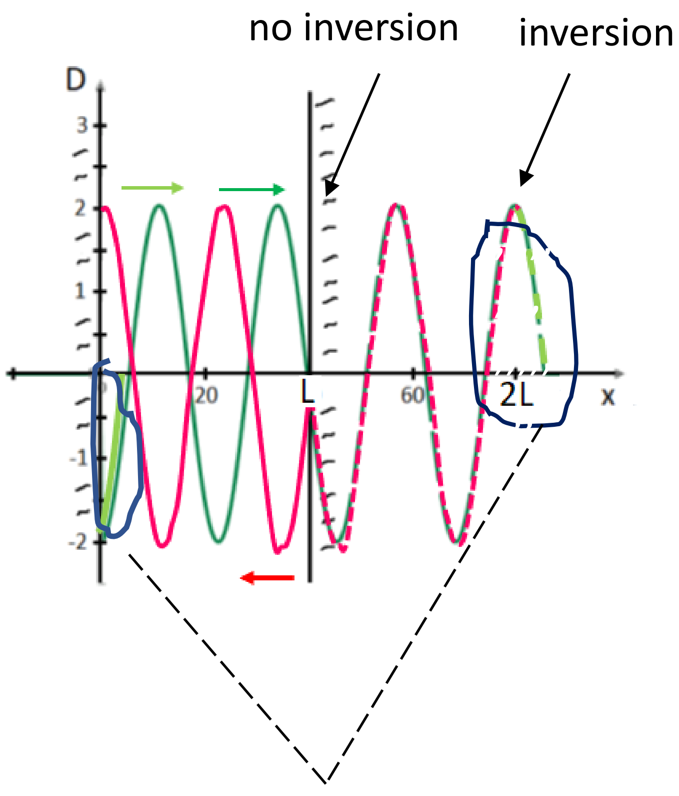
Note the presence of **nodes**, and that there is one at the hard boundary.

And note again ***antinodes***, especially the one at the soft boundary. There is always an antinode at a soft boundary.



D.2 Single Source Interference: Standing Waves

That's one particular example of resonance, but now let's analyze stuff in more generality. Say we have a clarinet of length, L. Which wavelengths will resonate? Again, if we look back through the previous illustration, we'll see that we'll get resonance automatically, if the 2nd reflected wave is in phase with the original incident wave. So we need,



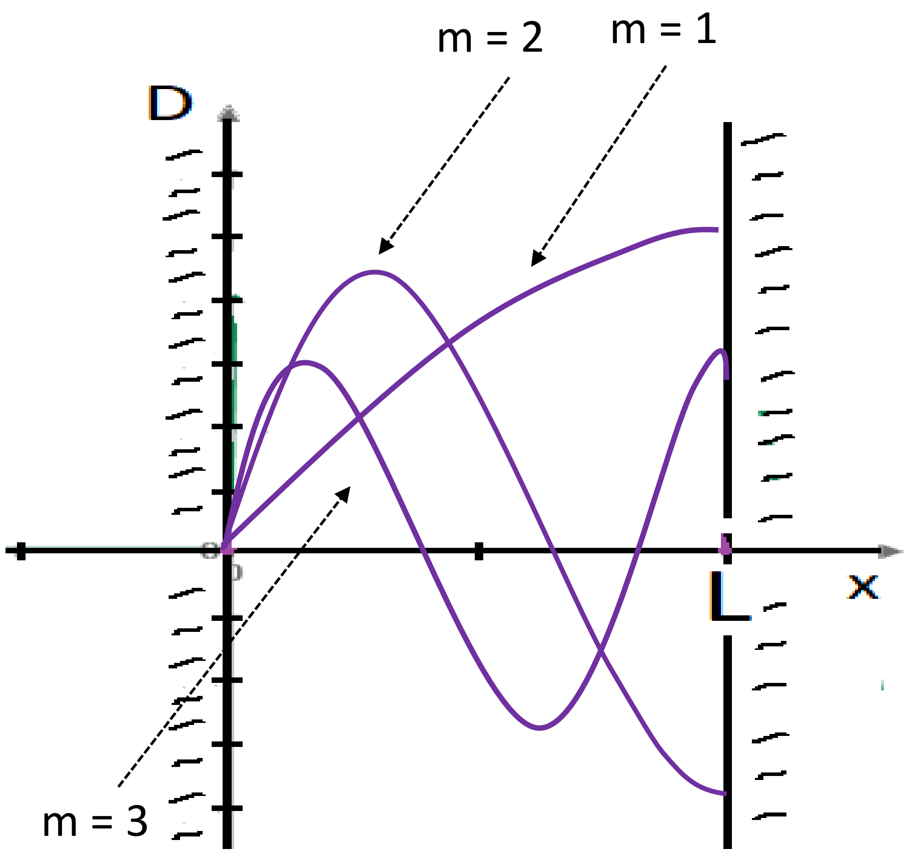
$$\Delta\varphi = 2\pi m \qquad m = 0, \pm 1, \pm 2, \text{etc.}$$

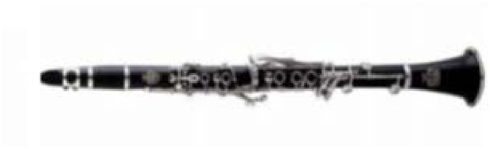
$$k\Delta x + I\pi = 2\pi m$$

$$\frac{2\pi}{\lambda}(2L) + 1\cdot\pi = 2\pi m$$

$$\lambda = \frac{2L}{m - 1/2}$$

- only m ≥ 1 makes sense, is # of nodes
- m = 1 gives is 'fundamental', or 'first' harmonic
- m = 2, 3, 4 etc. are 2nd, 3rd, 4th, etc. harmonics





D.2 Single Source Interference: Standing Waves

We have a clarinet with all stops (is that what they're called?) closed, except for one 40cm from the reed.
Say the air temperature is T = 300K, and its molar mass is 0.028kg. What are its first three resonant frequencies?

Repeating our analysis, the resonant waveforms are given by:

$$\Delta\varphi = 2\pi m$$

$$k\Delta x + I\pi = 2\pi m$$

$$\left(\frac{2\pi}{\lambda}\right)(2\times 40\text{cm}) + \pi = 2\pi m$$

$$\lambda = \frac{80\text{cm}}{m - 1/2} \qquad m \geq 1$$

So the first three resonant wavelengths are:

$$\lambda = \frac{80\text{cm}}{(1,2,3) - 1/2} = 160\text{cm}, 53\text{cm}, 32\text{cm}$$

then to get the resonant frequencies we just use:

$$f = \frac{v}{\lambda}$$

but what's v?

$$v = \sqrt{\frac{\gamma RT}{m_{molar}}} = \sqrt{\frac{(1.4)(8.31)(300)}{0.028}} = 353\,\text{m/s}$$

therefore,

$$f = \frac{353}{1.60}, \frac{353}{0.53}, \frac{353}{0.32} = 221\text{Hz},\ 666\text{Hz},\ 1100\text{Hz}$$

'fundamental' frequency

higher harmonics are *odd* multiples of fundamental

D.2 Single Source Interference: Standing Waves

Which frequency do you hear when you blow on the end of the clarinet?

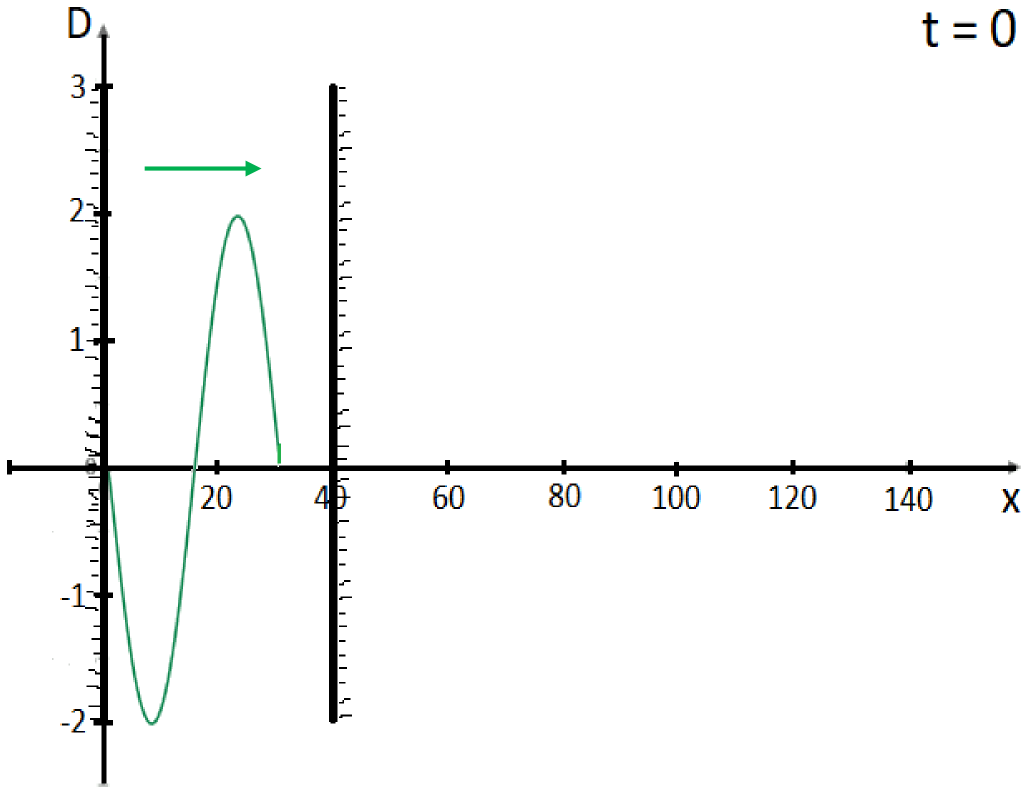
You'll hear the one whose waveform whose frequency most closely matches the frequency of the sound wave you're creating when you buzz on the end of the reed. Often this will be the fundamental frequency, but if you 'buzz' faster then you can reach the next couple harmonics.

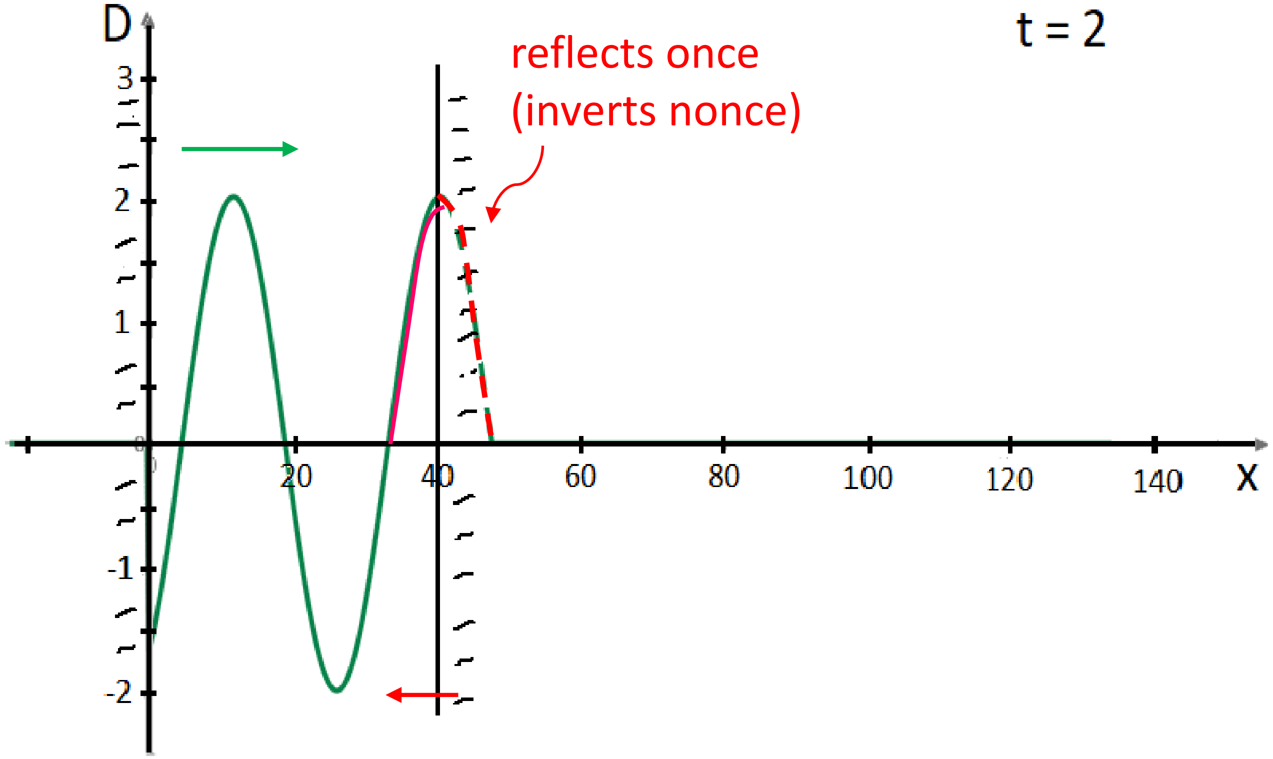
What would happen to these frequencies if the air temperature increased, or if you played in a room full of helium instead of air (never mind how you'd survive)?

Increasing T would increase v, and so would increase f. Switching out air for He would decrease m_{mol} which would also increase v, increasing f.

D.2 Single Source Interference: Standing Waves

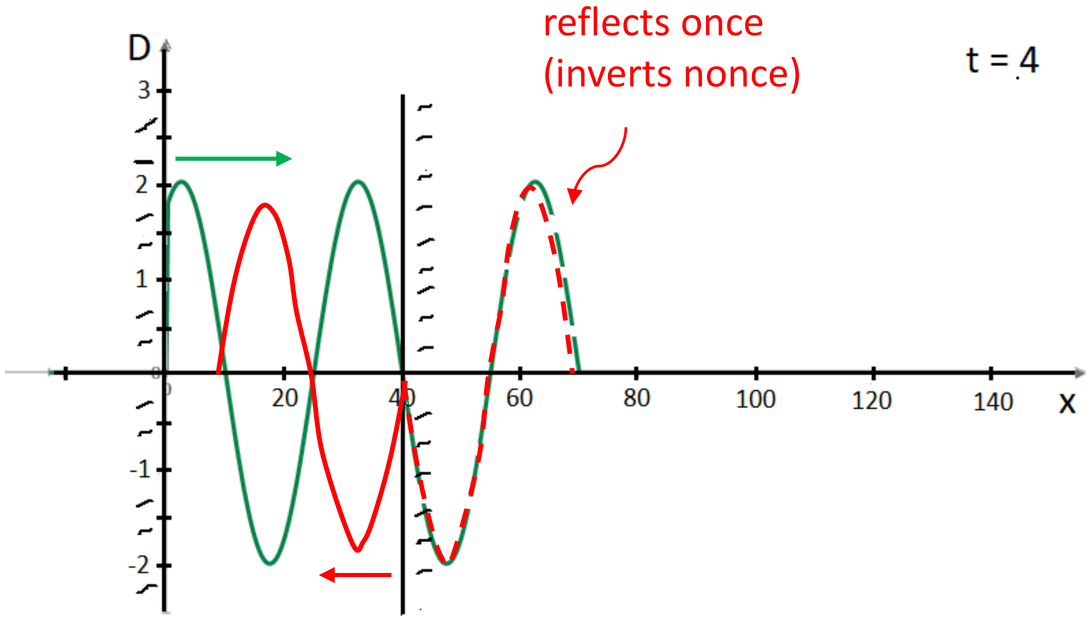
Last, let's consider the flute example. The flute is played with your mouth just above the opening, which effectively makes this a soft boundary for the air column. So a flute would be considered to have a soft boundary on either end. Thus, the waves which reflect back and forth along the air column will not invert at all. Let's consider a 30cm wave first, traveling down our 40cm long flute.

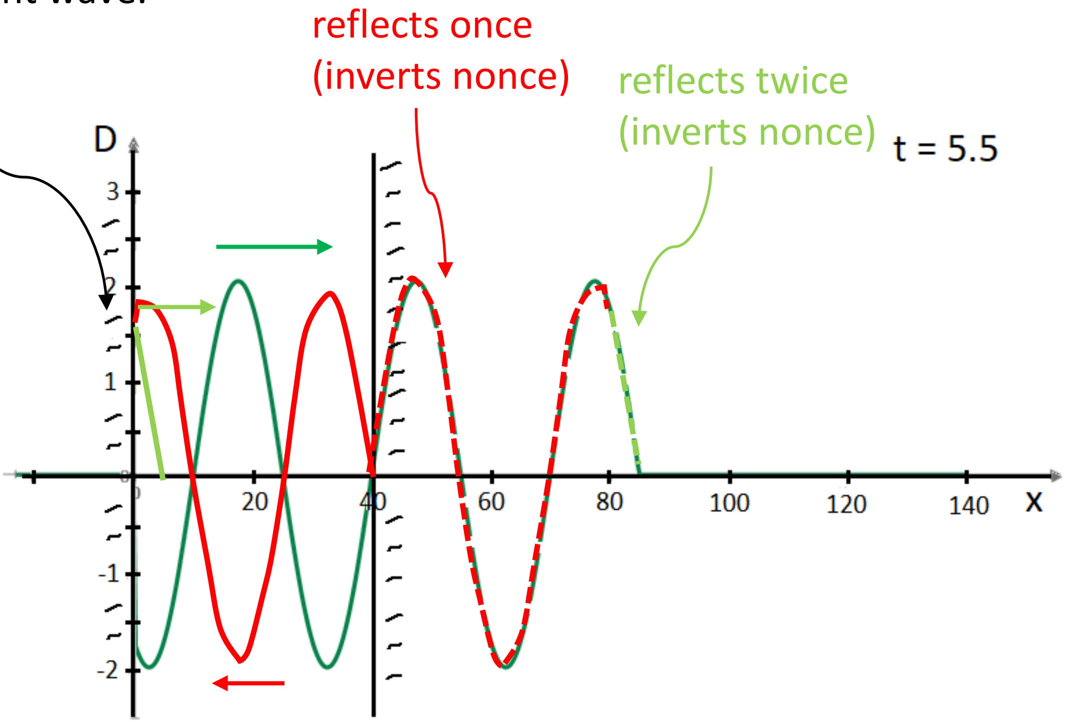




D.2 Single Source Interference: Standing Waves

already going bad, because this rightward going wave is not overlapping the original incident wave.

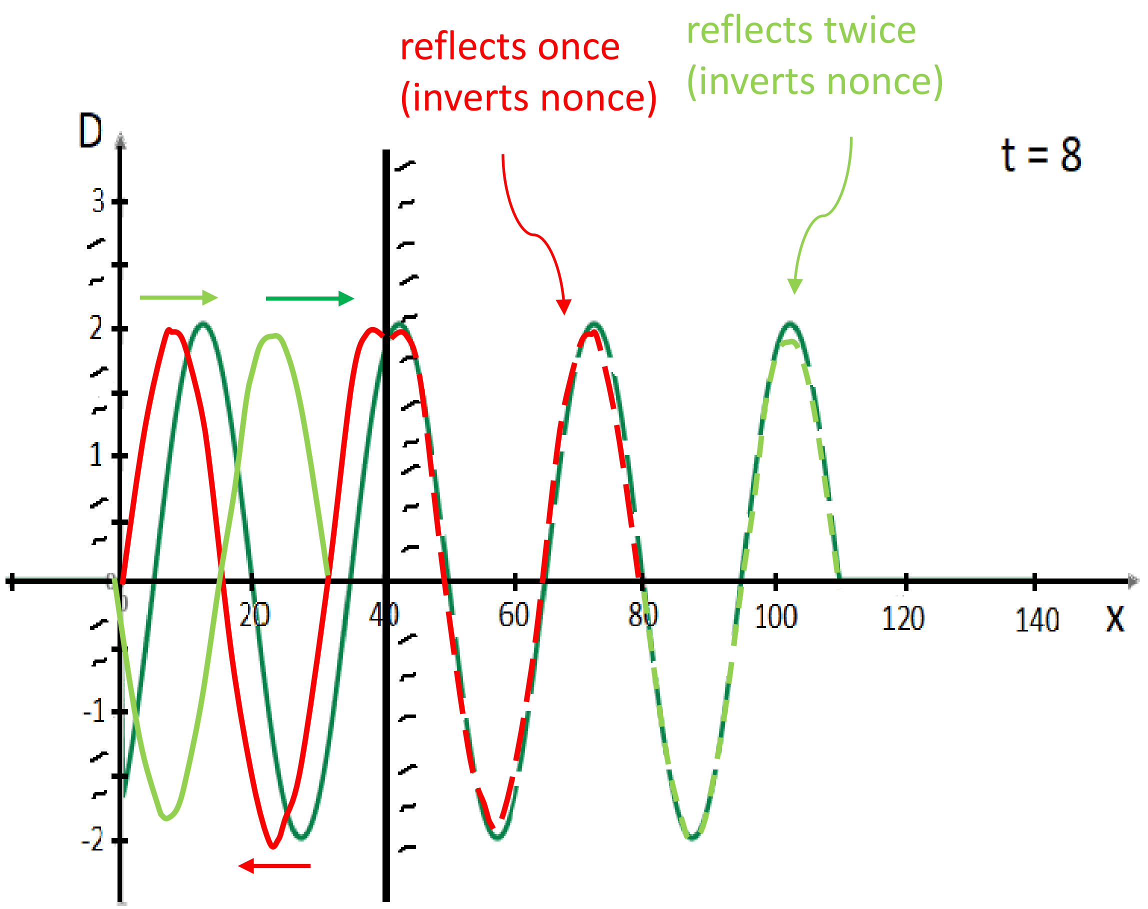


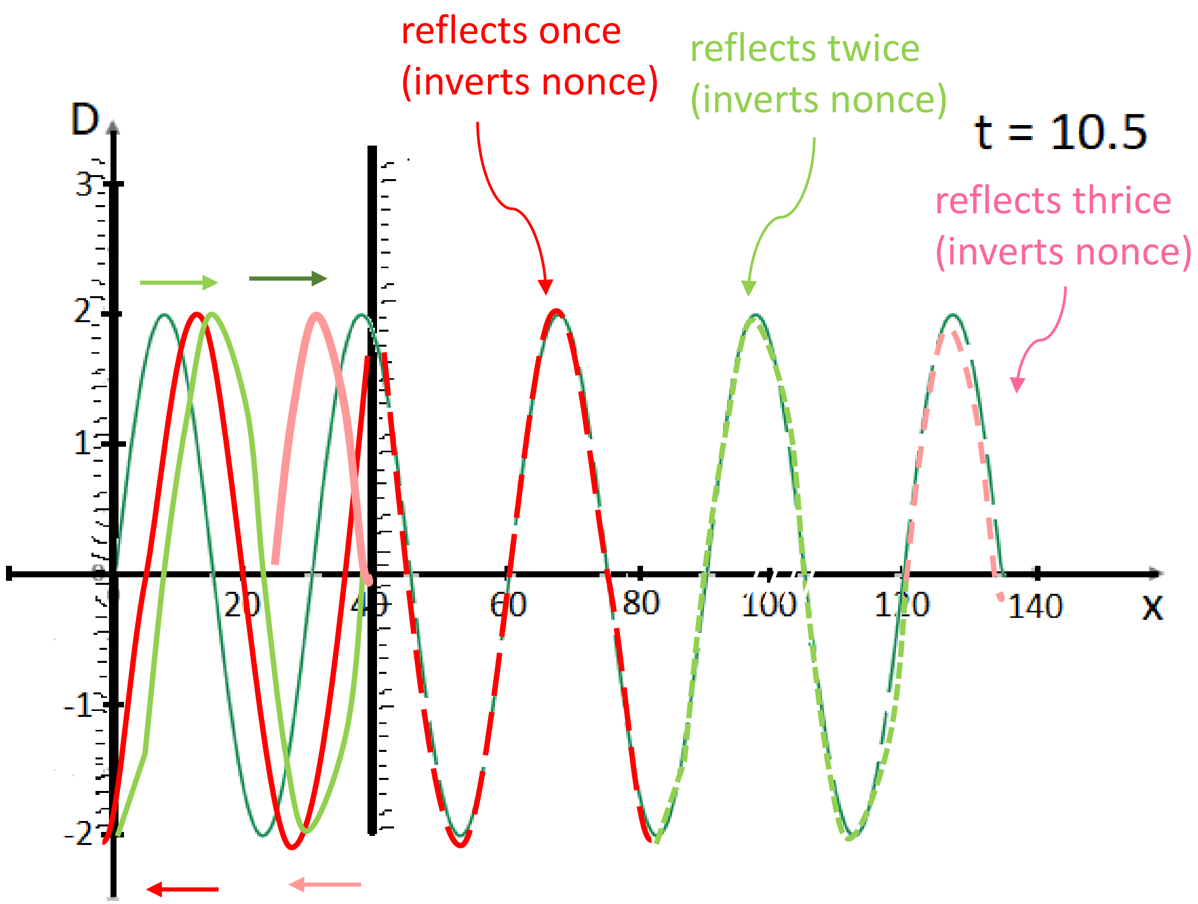


D.2 Single Source Interference: Standing Waves

Still bad

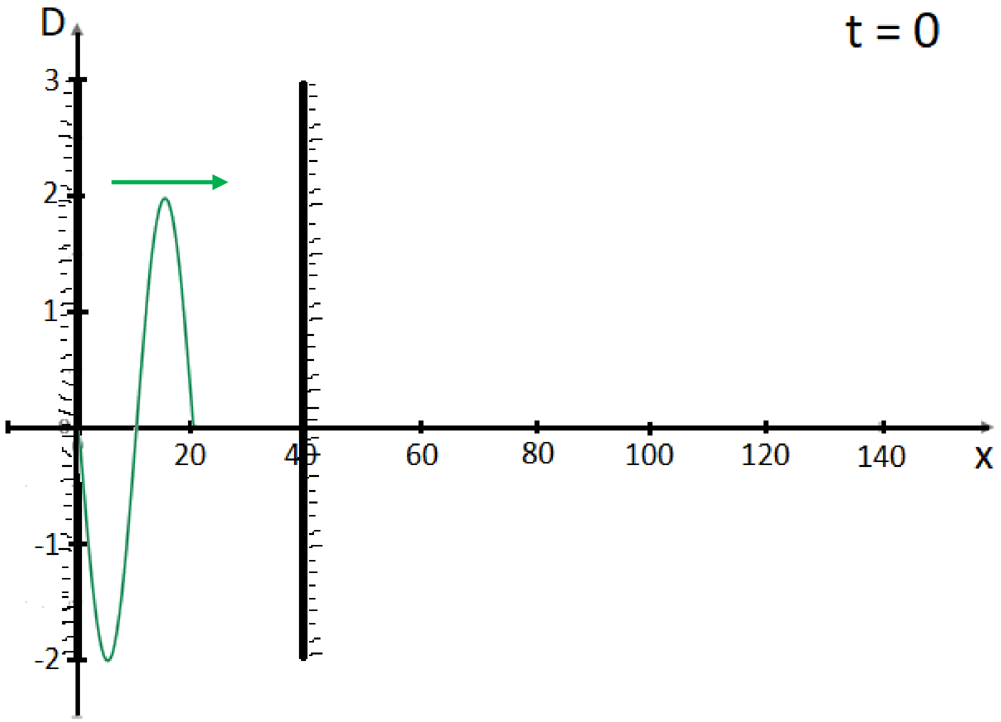
Yep, still bad

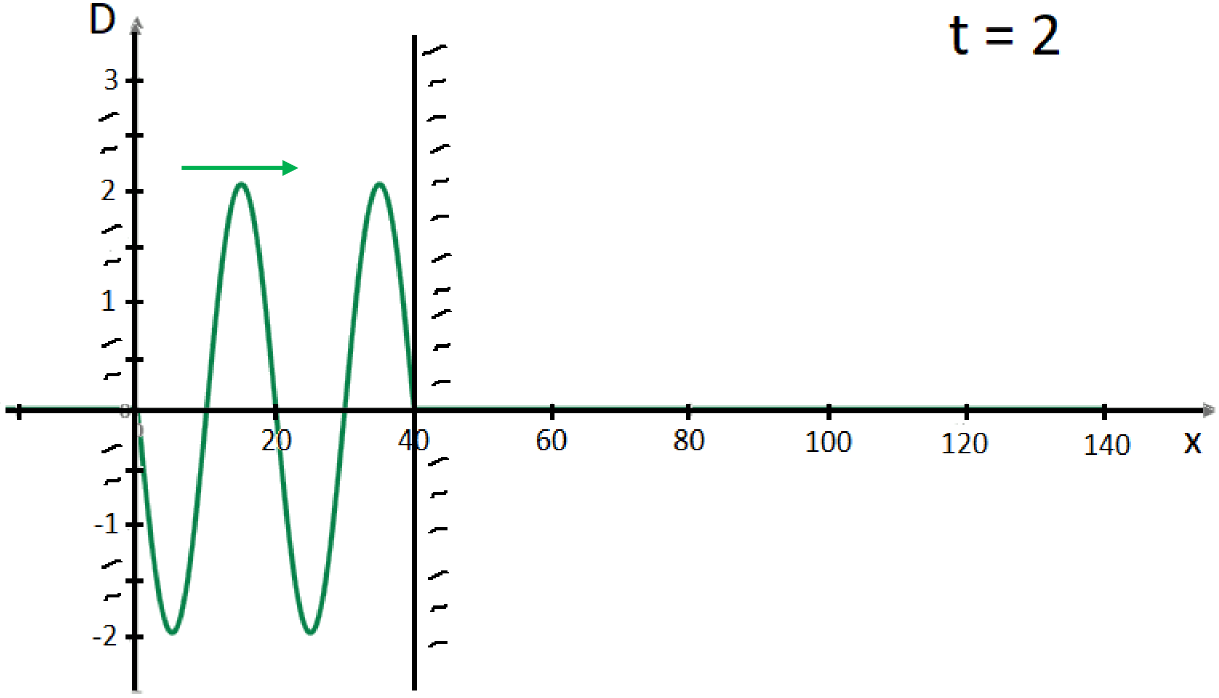




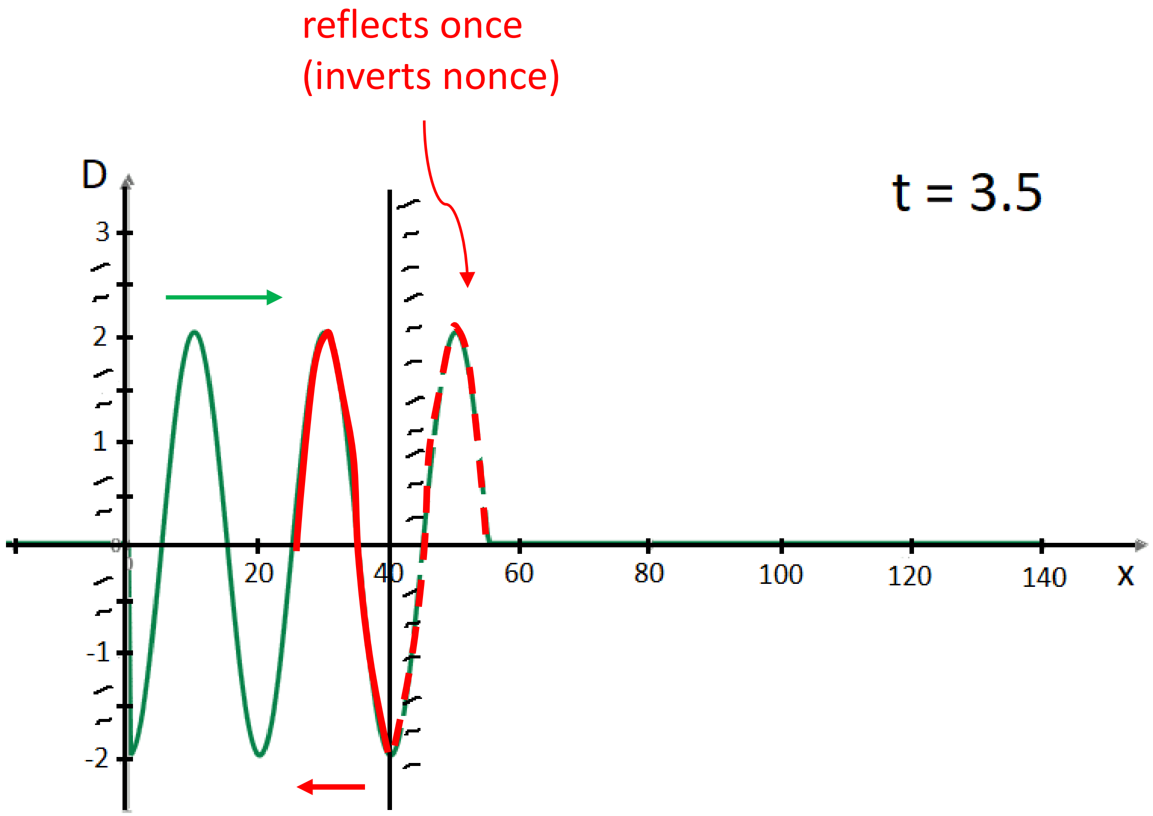
D.2 Single Source Interference: Standing Waves

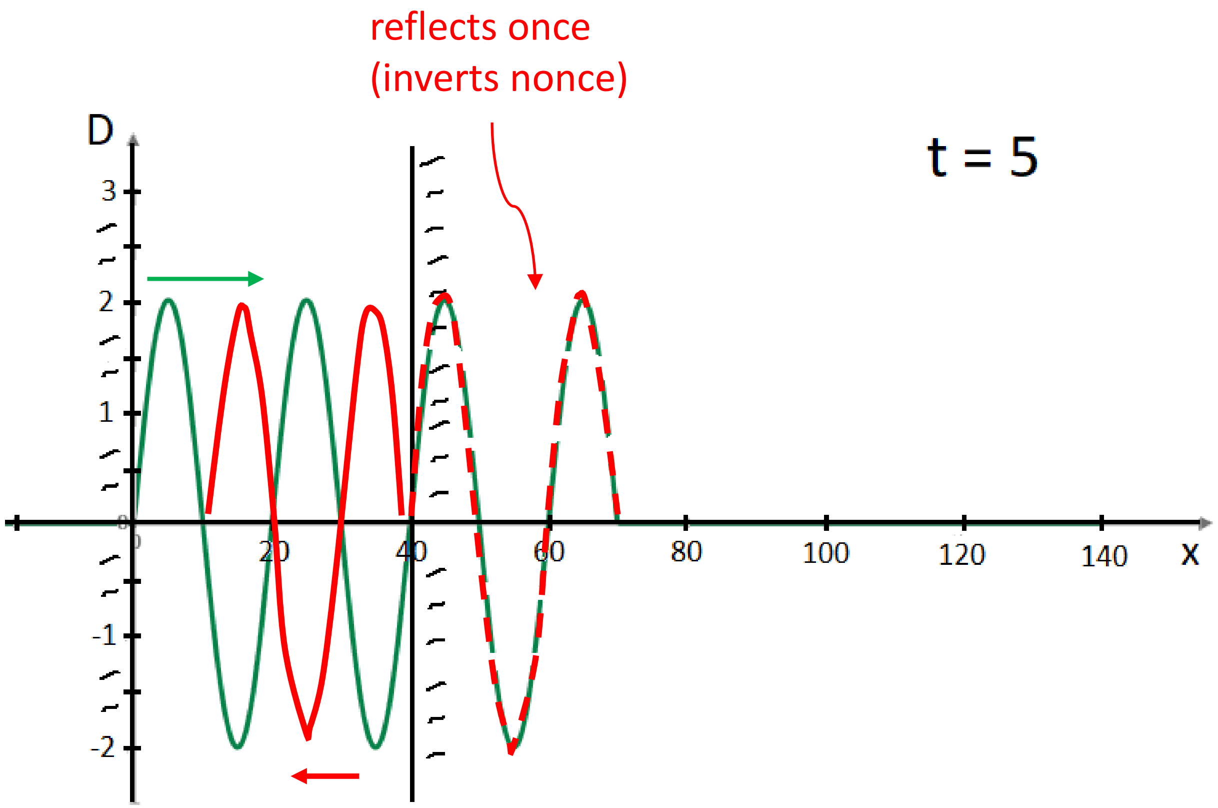
But there are (surprise!) cases where we do get resonance. Consider a λ = 20cm wave, traveling at 10cm/s as well.





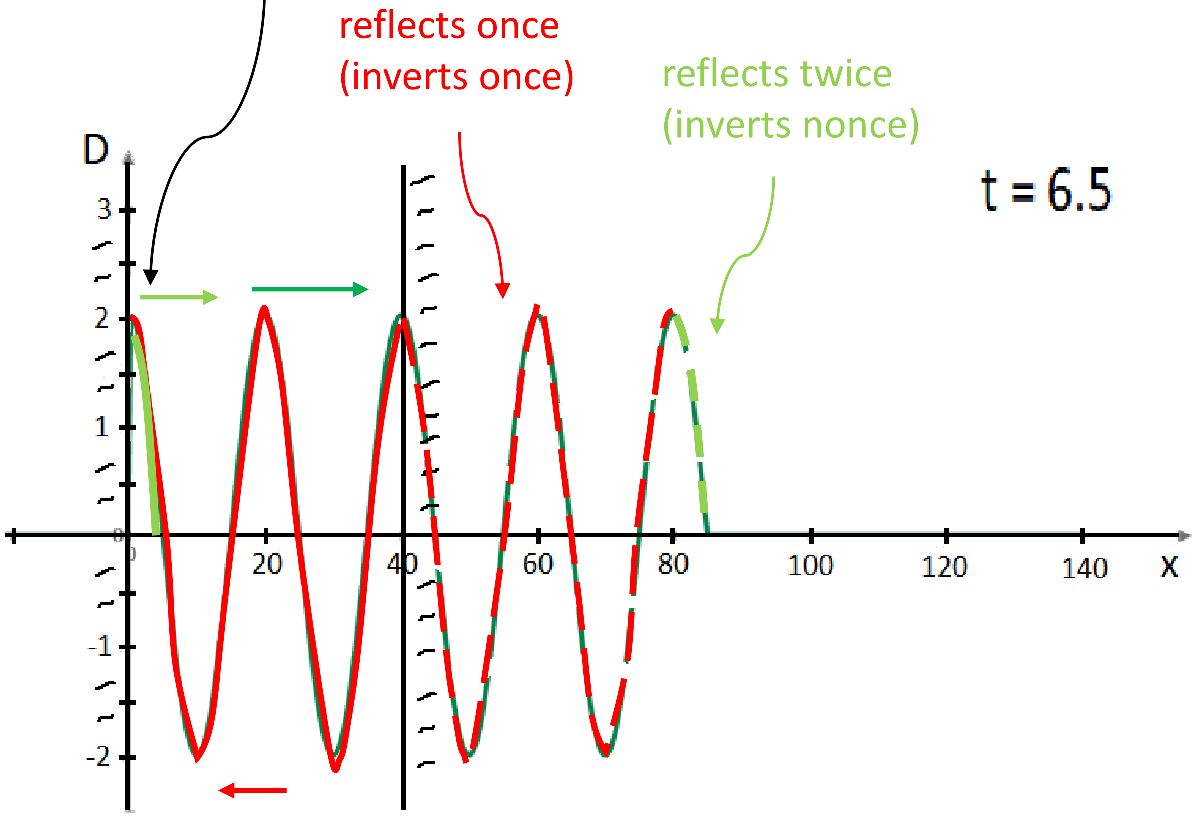
D.2 Single Source Interference: Standing Waves



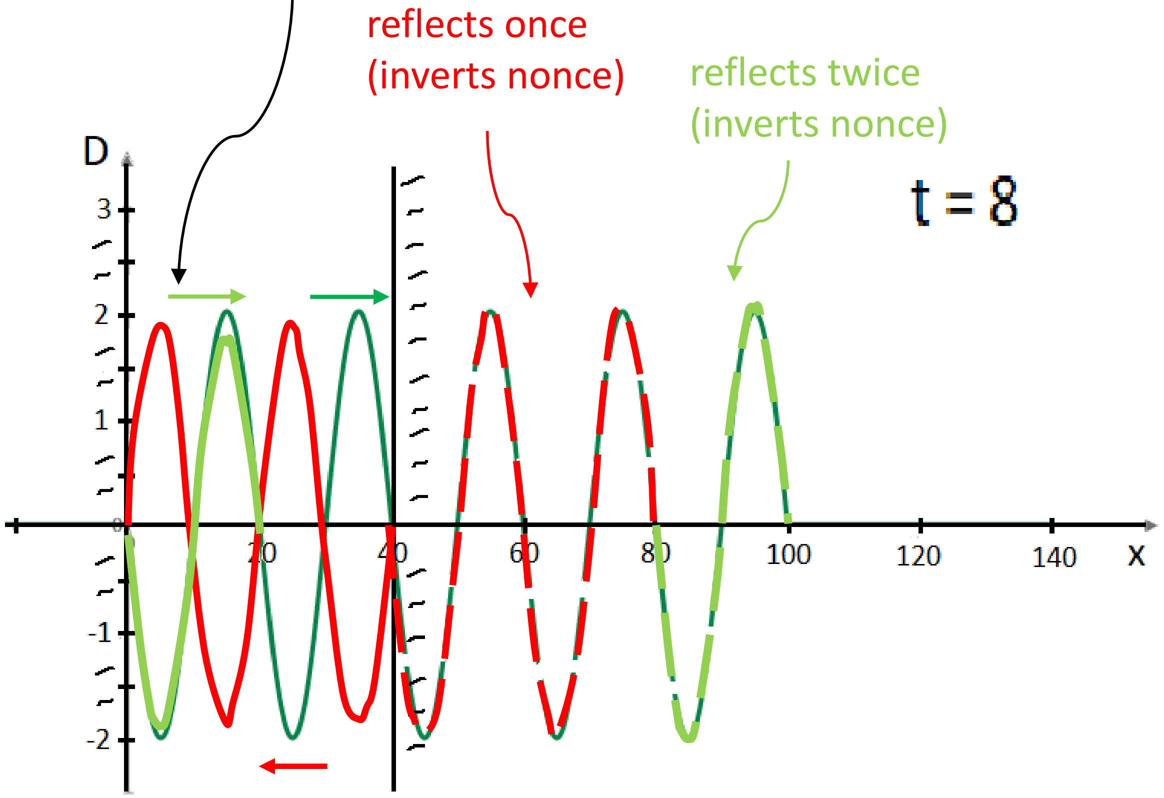


D.2 Single Source Interference: Standing Waves

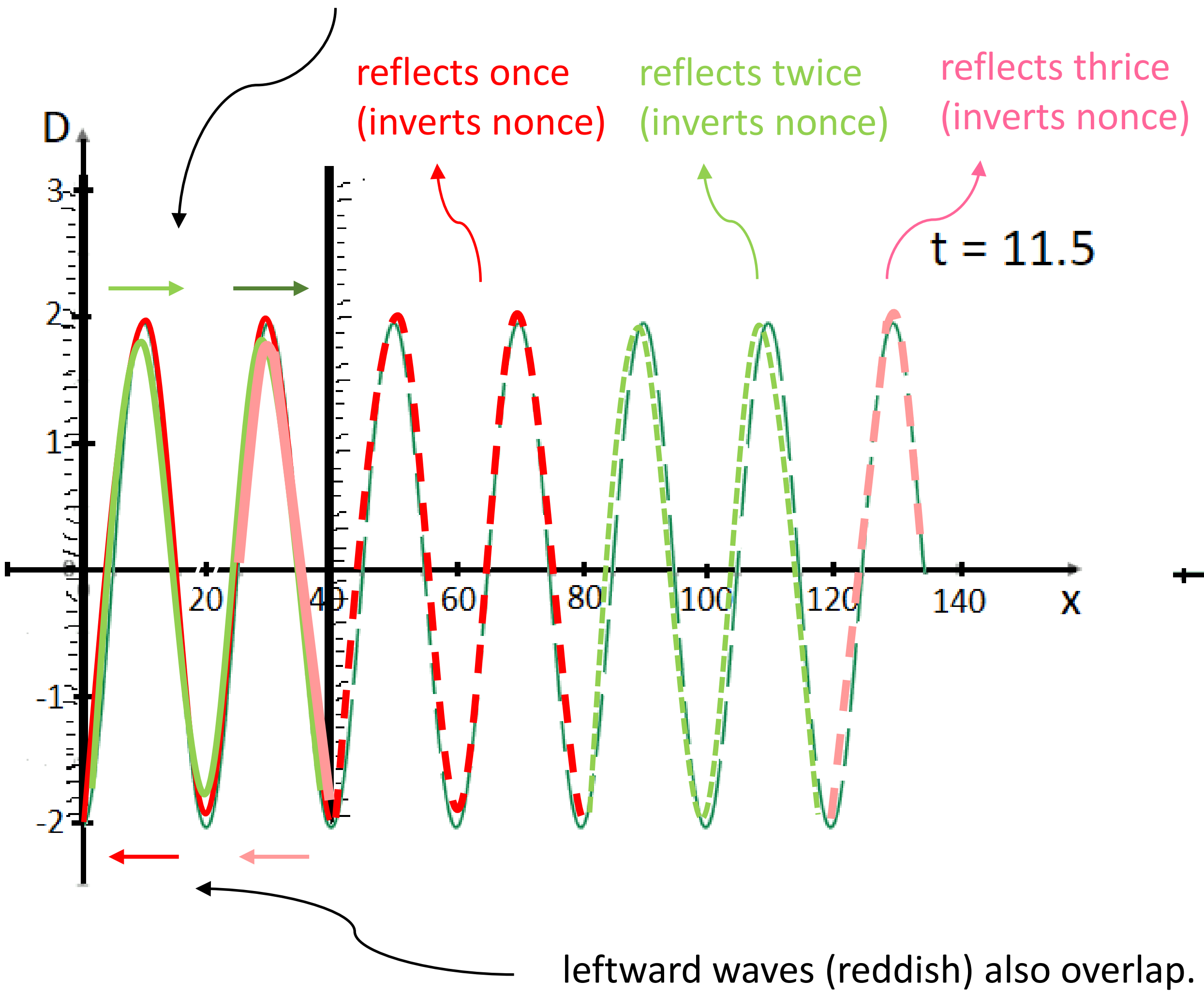
starting off well, because the two rightward waves (greenish) overlap

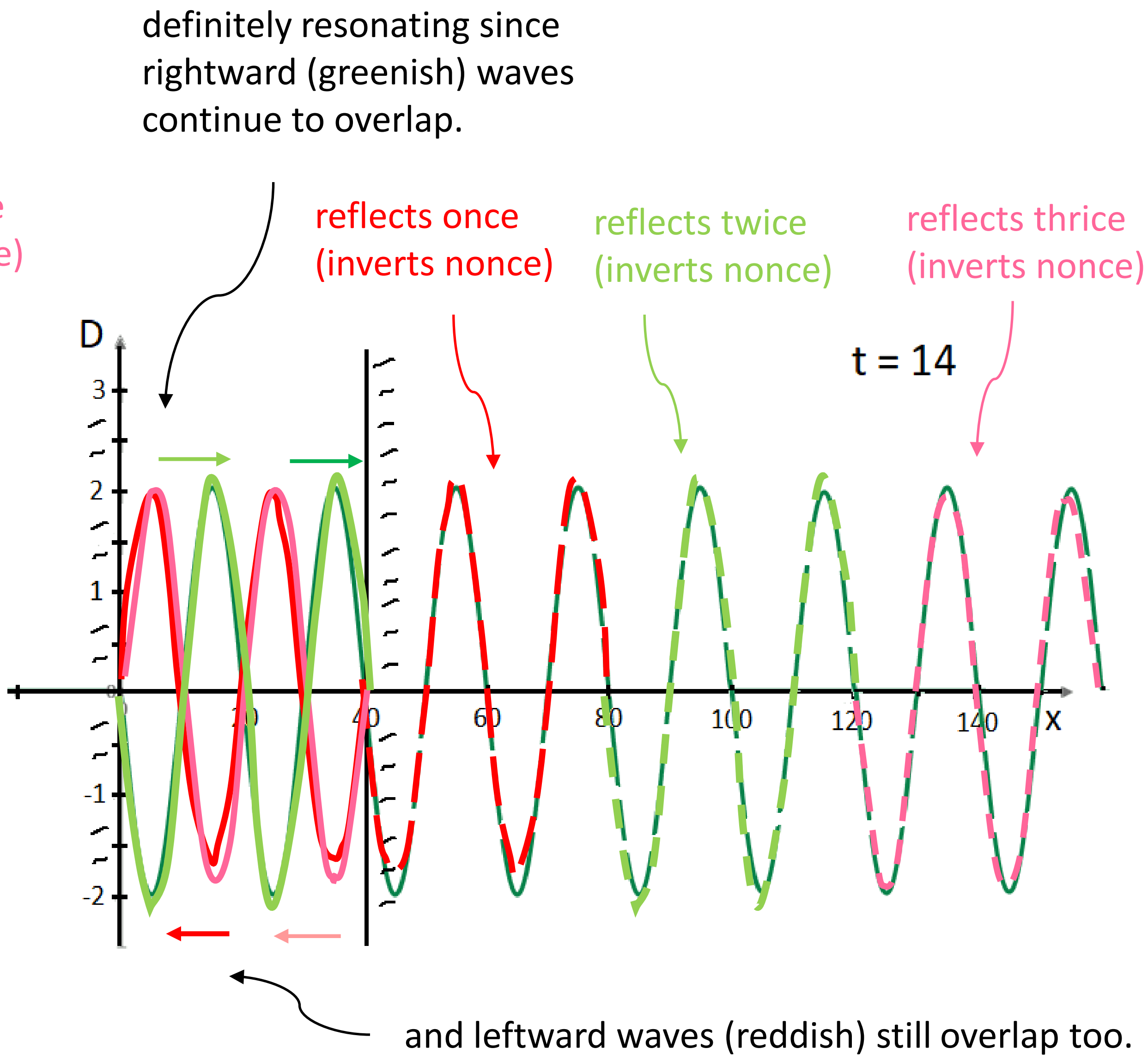


still going well because rightward (greenish) waves continue to overlap.



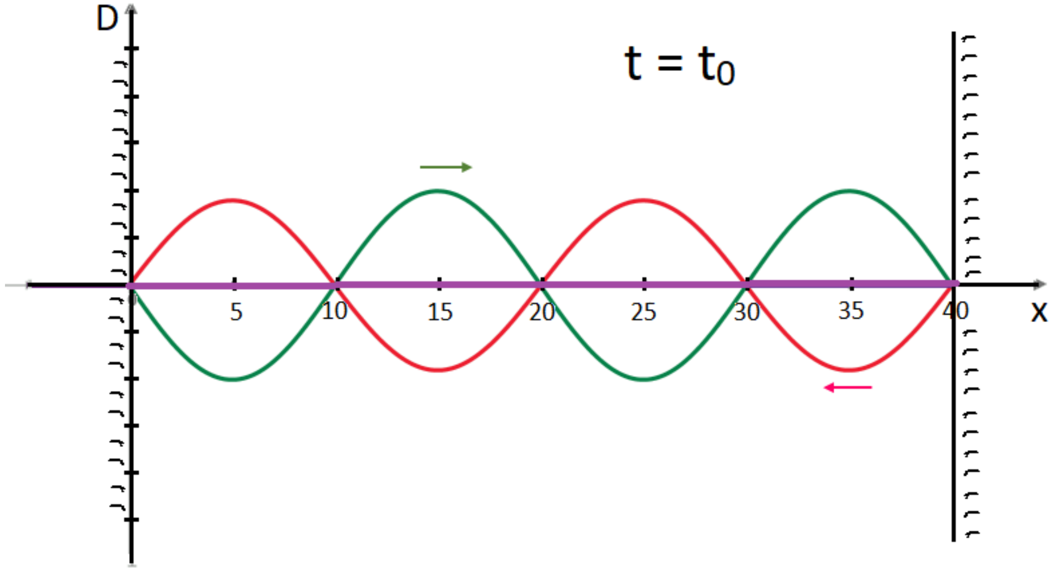
D.2 Single Source Interference: Standing Waves

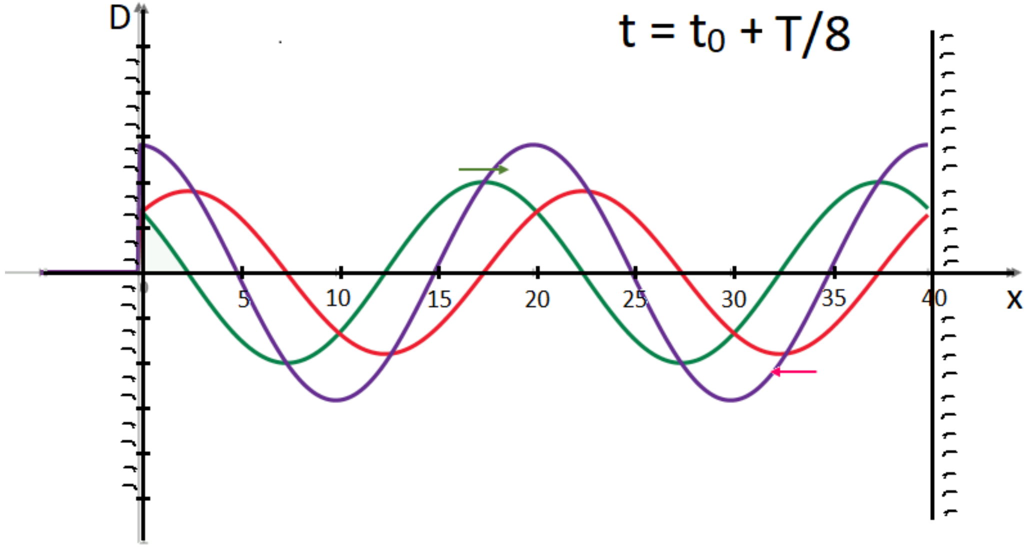




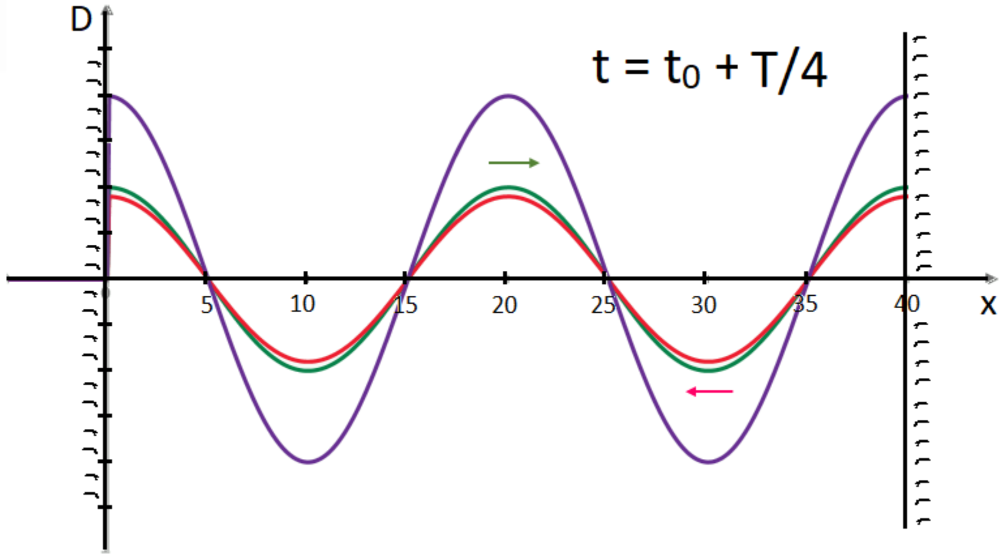
D.2 Single Source Interference: Standing Waves

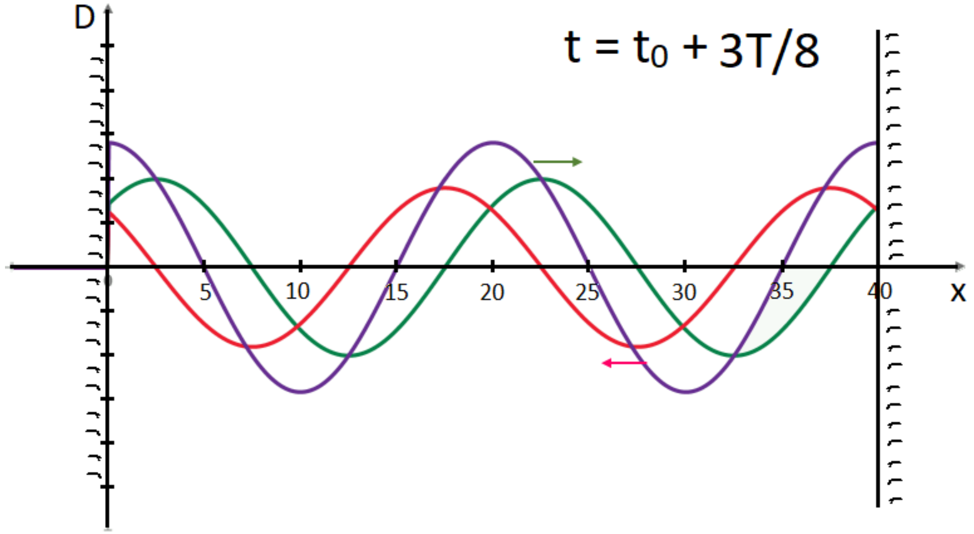
Now let's see what this resonant wave will look like on the string. Green is going right; red to the left, and purple is the superposition. I'll progress time in increments of an eight of period, starting from some time when the waves overlap, like they do at t = 14s.

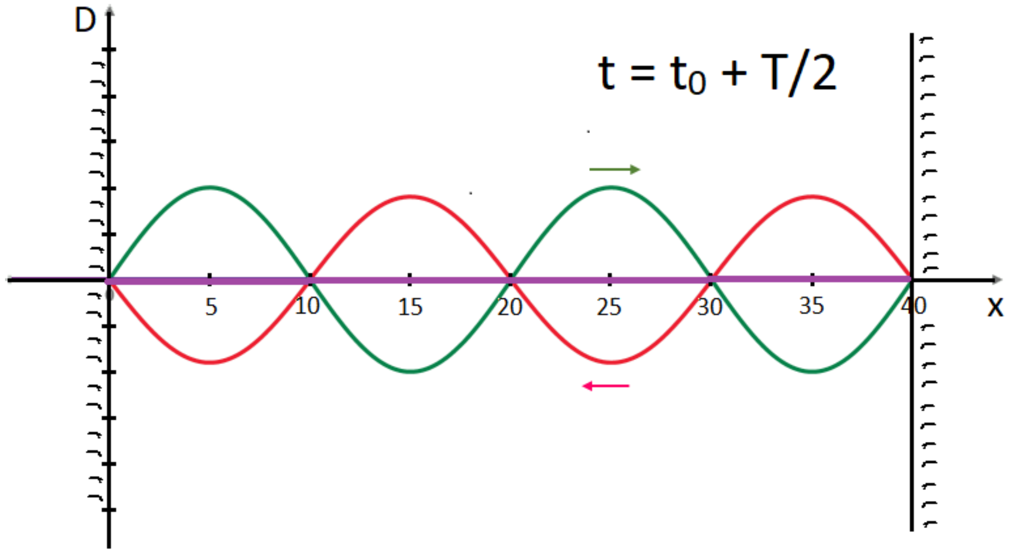


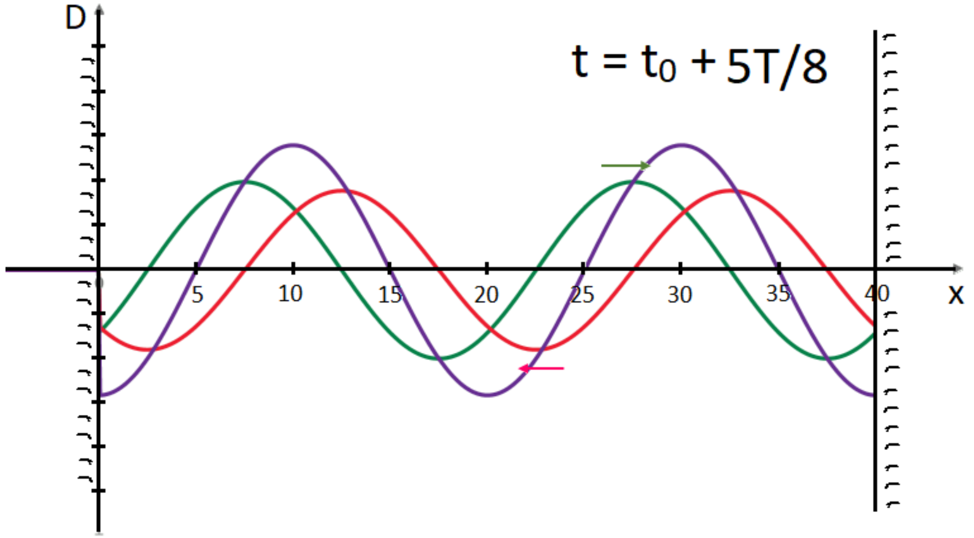


D.2 Single Source Interference: Standing Waves

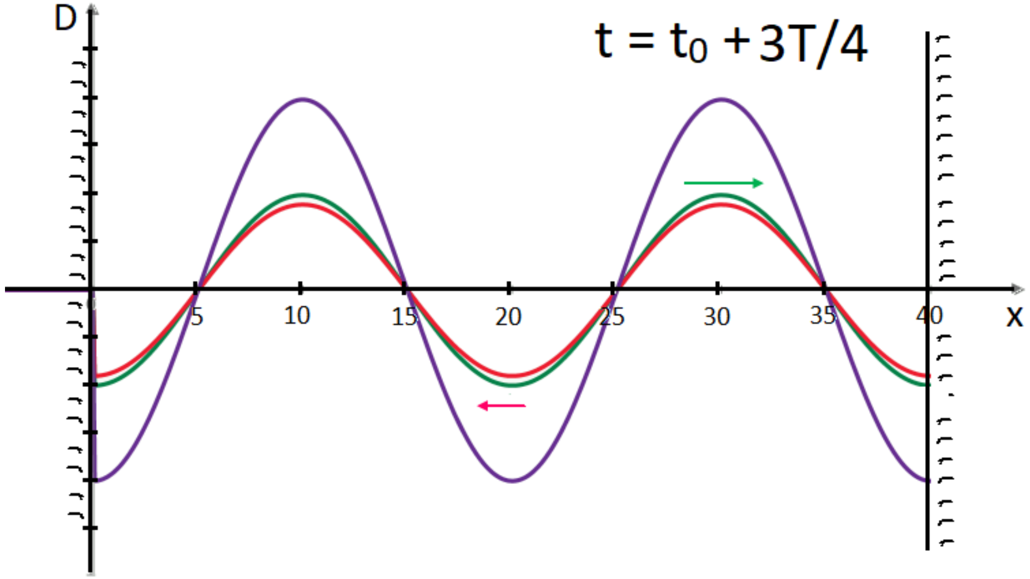


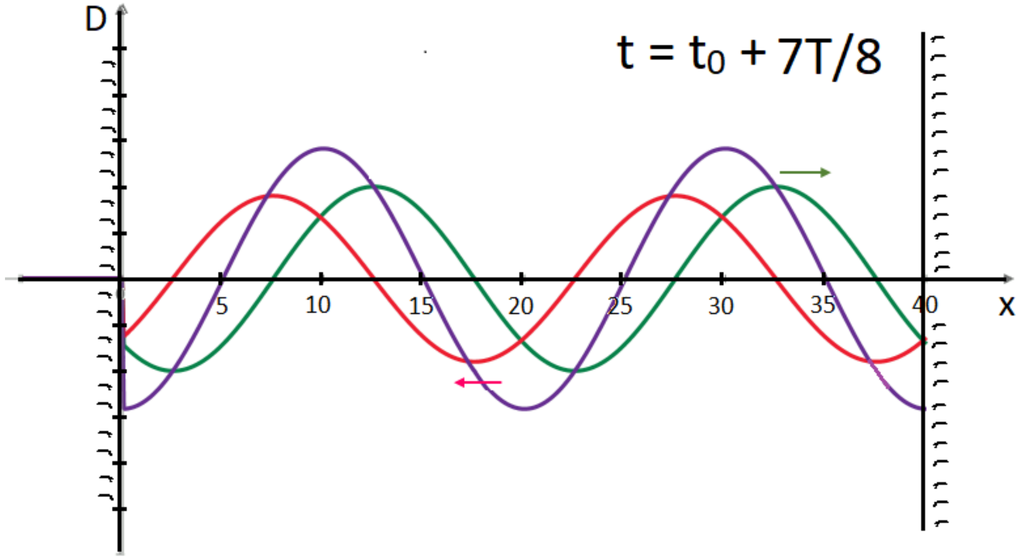






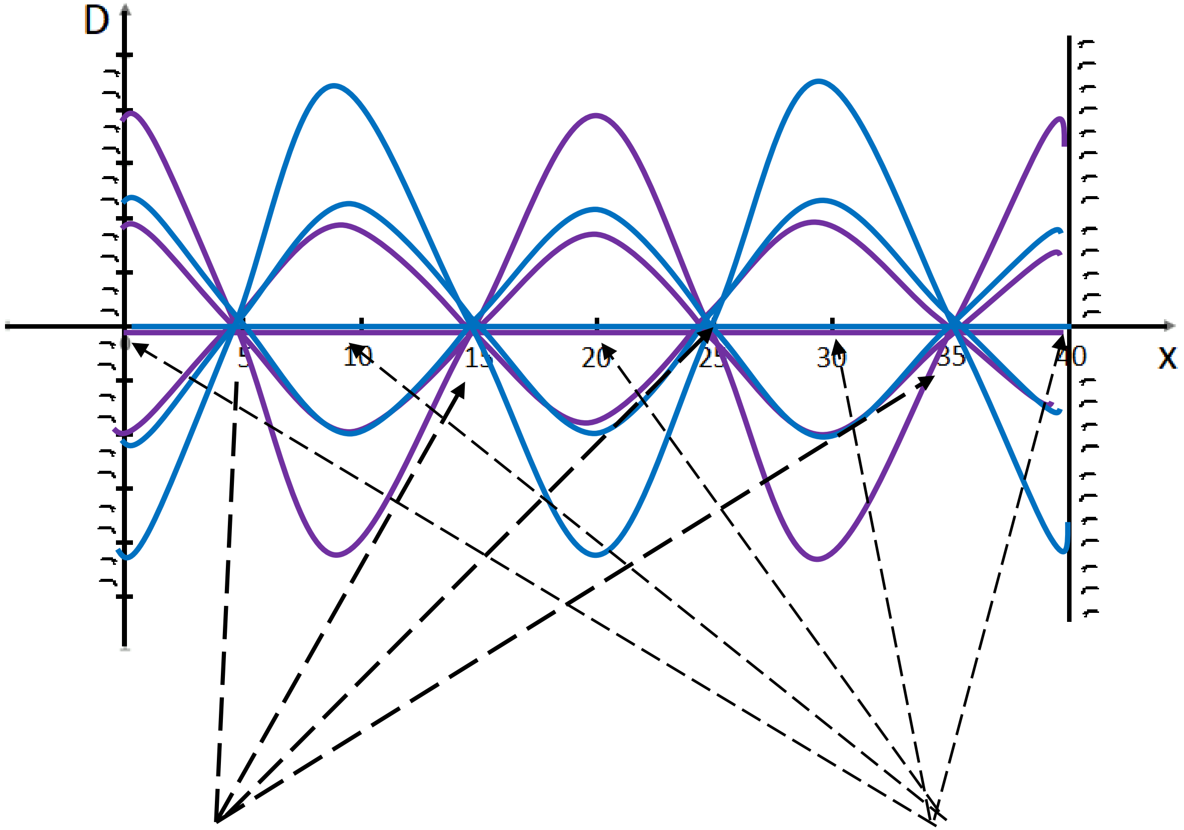
D.2 Single Source Interference: Standing Waves





D.2 Single Source Interference: Standing Waves

All at once, just drawing the superpositions, we get another standing wave which will repeat itself every period T.



Basic features again:

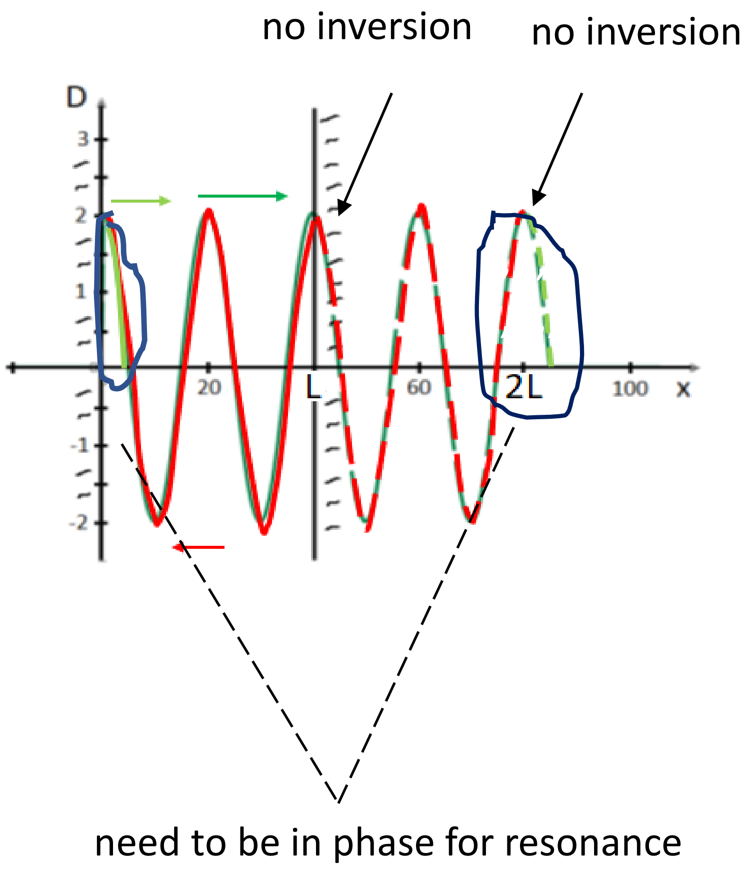
1. Same as shape of original wave.
2. Positioned so *nodes* are at any *hard* boundary (none here), and *antinodes* at any *soft* boundary (both).
3. As time proceeds, wave *maintains* basic shape, with a *highly* augmented amplitude, but its size alternately shrinks and grows with time with a period T equal to the original wave's own period. This superposition is called a standing wave.

Note the presence of **nodes**, and that there is none at the boundaries, because none are hard.

And note again ***antinodes***, especially the ones at the soft boundaries. There is always an antinode at a soft boundary.

D.2 Single Source Interference: Standing Waves

That's one resonance, but now let's analyze stuff in more generality. Say we have a flute of length, L. Which wavelengths will resonate? Again, if we look back through the previous illustration, we'll see that we'll get resonance automatically, if the 2nd reflected wave is in phase with the original incident wave. So we need,



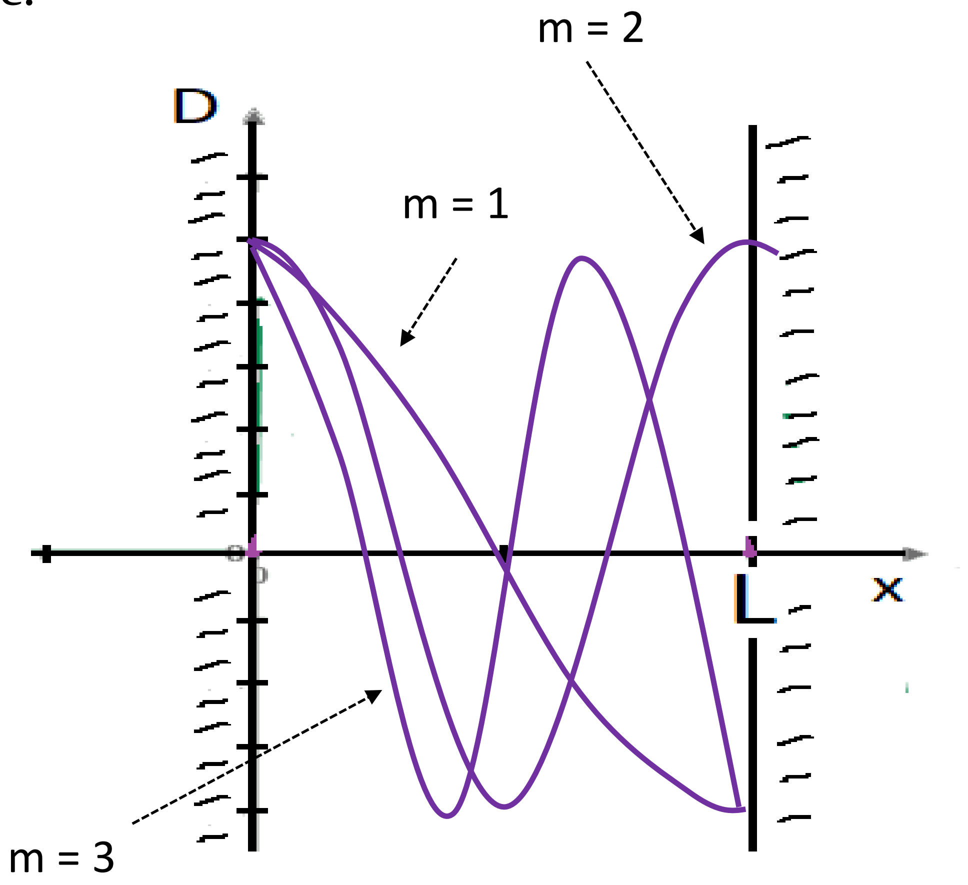
$$\Delta\varphi = 2\pi m \qquad m = 0, \pm 1, \pm 2, \text{etc.}$$

$$k\Delta x + I\pi = 2\pi m$$

$$\frac{2\pi}{\lambda}(2L) + 0\cdot\pi = 2\pi m$$

$$\lambda = \frac{2L}{m}$$

- only m ≥ 1 makes sense, is # of nodes
- m = 1 gives is 'fundamental', or 'first' harmonic
- m = 2, 3, 4 etc. are 2nd, 3rd, 4th, etc. harmonics



D.2 Single Source Interference: Standing Waves

Say we have all the stops closed on a flute, except for one 40cm down. Air temperature and molar mass are 300K and 0.028kg again. What would be the first three resonant frequencies?

Repeating our analysis, the resonant waveforms are given by:

$$\Delta\varphi = 2\pi m$$

$$k\Delta x + I\pi = 2\pi m$$

$$\left(\frac{2\pi}{\lambda}\right)(2\times 40\text{cm}) + 0\cdot\pi = 2\pi m$$

$$\lambda = \frac{80\text{cm}}{m} \qquad m \geq 1$$

So the first three resonant wavelengths are:

$$\lambda = \frac{80\text{cm}}{(1,2,3)} = 80\text{cm}, 40\text{cm}, 27\text{cm}$$

then to get the resonant frequencies we just use:

$$f = \frac{v}{\lambda}$$

but what's v?

$$v = \sqrt{\frac{\gamma RT}{m_{molar}}} = \sqrt{\frac{(1.4)(8.31)(300)}{0.028}} = 353\,\text{m/s}$$

therefore,

$$f = \frac{353}{0.80}, \frac{353}{0.40}, \frac{353}{0.27} = 441\text{Hz, } 882\text{Hz, } 1223\text{Hz}$$

'fundamental' frequency

higher harmonics are multiples of fundamental

D.2 Single Source Interference: Standing Waves

Which frequency do you hear when you blow on the end of the flute?

You'll hear the one whose waveform whose frequency most closely matches the frequency of the sound wave you're creating when you blow on the mouthpiece part.   Often this will be the fundamental frequency, but if you 'blow' faster then you can reach the next couple harmonics.

What would happen to these frequencies if the air temperature increased, or if you played in a room full of helium instead of air?

Increasing T would increase v, and so would increase f. Switching out air for He would decrease m_{mol} which would also increase v, increasing f.